

## Newton in the Underworld<sup>1</sup>

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### Introduction

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The following notes provide a glimpse into some of the early attempts to apply mechanics to engineering problems involving rock *in situ*. In many instances, the engineers had little or no research support, and in attempting to use results developed for other applications, quickly learned that practical situations were more complex than when dealing with fabricated materials with well-defined properties, as assumed in then standard texts on mechanics. The First US Symposium on Rock Mechanics, co-sponsored by three US Mining schools held sixty years ago, in April 1956, was sponsored by three university departments of Mining Engineering (Colorado School of Mines; Minnesota; Pennsylvania State) but included participants from Civil and Petroleum Engineering groups. Clearly, attendees were there to raise questions, and to highlight the need for research. Other parts of the world were also active. The National Coal Board (UK), established in 1945, immediately after WWII, had research laboratories investigating a variety of topics including mine-induced surface subsidence. [Berry and Sales (1962)]. France and Germany had Mining and Civil Engineering research groups. Rockburst problems in the deep gold mines of South Africa<sup>2</sup> and the Kolar Gold Field, India<sup>3</sup> were severe [Krishnamurthy and Shringarputale (1990)] The Chamber of Mines [led by Cook and Salamon] and CSIR (Pretoria) [led by Hoek, and Bieniawski] were making major contributions. The papers by Cook (1965) Cook et al; (1966); Durrheim (2010) Salamon (1974) and Ryder(1988) are examples of the international leadership of South African colleagues in mining research at that time. The paper by Krishnamurthy and Shringarputale (1990) describes the parallel efforts of the Kolar group to address the rockburst hazard.

By 1966, at the First ISRM Congress in Lisbon, rock mechanics was mobilizing internationally and beginning to define the special characteristics of rock *in situ*. It was here that President Leopold Müller, in his Opening Address to the Congress, stressed the importance of 'discontinuities and anisotropy' [see *Consideration of Discontinuities in a Rock Mass* below] as distinguishing Rock Mechanics from Soil Mechanics and classical Continuum Mechanics. The Third ISRM Congress in Denver, Sept 1-7,(1974) the only Congress ever held in the US, included five main themes. 1-Physical Properties of Intact Rock and Rock Masses. General Reporter J.Bernaix (France); 2. Tectonophysics; A.Nur (USA); 3. Surface Workings; E. Hoek (UK) and P. Londe, (France); 4. Underground Openings ; M.D.G. Salamon; ( RSA); 5. Fragmentation Systems C. H. Johansson (Sweden). The General Reports provide an excellent summary of the State of the Art of Rock Mechanics, internationally, in the early 1970's.

Although numerical methods were beginning to be applied in rock mechanics in the mid-1960's, [Stagg and Zienkiewicz (1968)], considerable reliance was still placed on various physical modeling techniques, and even on some early solutions from classical continuum analysis.

The following discussion describes some of these applications and the practical insights they offer, their extension through numerical modeling, including the introduction of discontinuities and analysis of jointed rock masses using physical models. Petroleum engineering applications to hydraulic fracturing included consideration of the mechanics of crack propagation due to the action of fluid under pressure. Application of scaling analysis to this problem provides valuable guidance on the relative importance of field variables on fracture development. Development and extension of fractures and fracture systems by fluid pressure is becoming a feature of a broader variety of subsurface engineering activities, so that the need to understand the processes involved increases correspondingly.

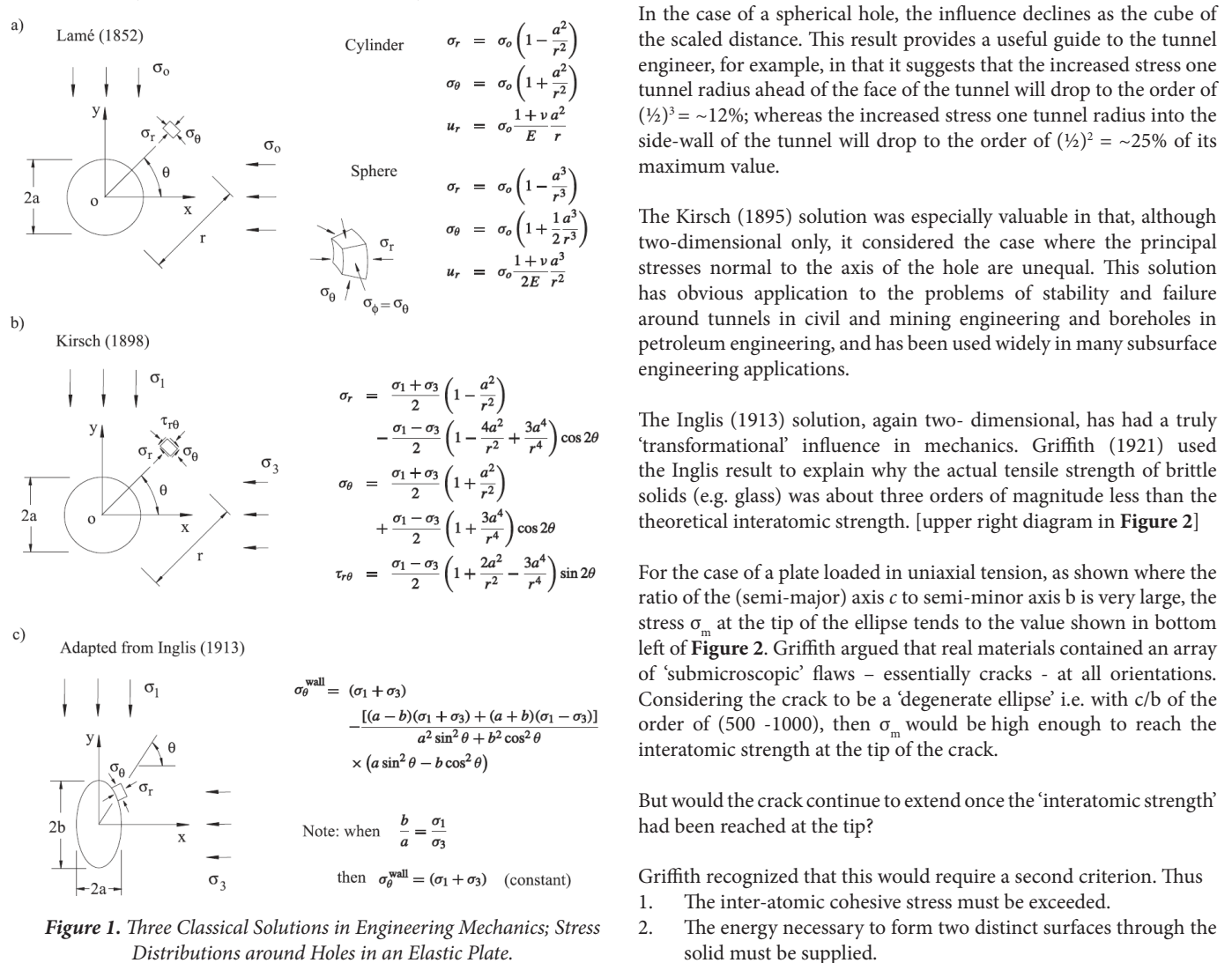
Despite these advances in the application of mechanics to rock engineering problems, the complex tectonic history and heterogeneity of rock *in situ* is such that some important feature affecting the rock behavior may be overlooked or represented incorrectly in the model. This suggests the value of subjecting any engineering project involving the behavior of rock *in situ* to review and scrutiny by professionals with a variety of backgrounds associated with rock. Several examples are given of situations where this team approach has proven beneficial

Introduction of the SubTER and FORGE programs by the US Dept of Energy is applauded. There remains now an urgent need to develop Engineering Research Centers in Earth (Subsurface) Resources Engineering at leading US research universities to ensure the continued development of this essential resource.

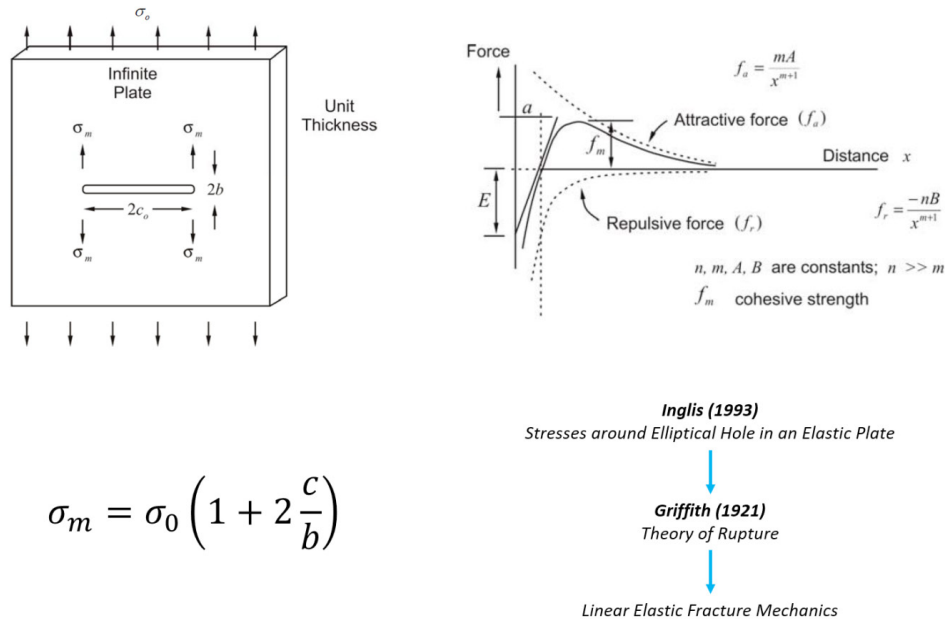
## Early Mechanics Analyses

Classical mechanics was a topic of discussion by Greek philosophers, but it was not until the arrival of the calculus (by Leibnitz and Newton, ca 1696) and the associated notion of the continuum<sup>4</sup> that developments of interest to engineers started to appear. Three early 'closed form' (continuum) solutions of especial significance to engineers are shown in **Figure 1**.

The Lamé (1852) solution indicates that the magnitude of change in stress around a circular hole in an elastic medium reaches a maximum at the boundary of the hole –but decreases away from the hole as the square of the distance from the hole –scaled to the radius of the hole.



**Figure 1. Three Classical Solutions in Engineering Mechanics; Stress Distributions around Holes in an Elastic Plate.**



**Figure 2.** The Strength of Brittle Materials –Griffith (1921).

To satisfy the second condition he invoked the Theorem of Minimum Potential Energy – “The stable equilibrium state of a system is that for which the potential energy of the system is a minimum” For the particular application of this theorem to rupture, Griffith added the statement

*“The equilibrium position, if equilibrium is possible, must be one in which rupture of the solid has occurred, if the system can pass from the unbroken to the broken condition by a process involving a continuous decrease of potential energy.”*

From these two principles, Griffith was able to provide an explanation of how brittle materials fail –in situations where the applied stresses are predominantly tensile.

The major field of Linear Elastic Fracture Mechanics (LEFM) developed from this basis, and has stimulated major advances in the design of high performance, lightweight materials; advances that continue to the present day.

### Application of Griffith Theory to Situations where Compression Dominates

In a second paper, Griffith (1924) considered the case of failure around cracks in a compressive stress field. He determined, incorrectly, that the uniaxial compressive/tensile strength ratio should be 8:1. Surprisingly, he failed to invoke his second criterion, based on the Theorem of Minimum Potential Energy. Had he done so, he would have seen that propagation of individual cracks in a compressive stress field is not an unstable process. It is now well established that, while ‘crack’ extension in uniaxial compression of brittle rock does start to occur at a stress of the order of 8 times the tensile strength, the observed compressive strength is typically in the range of 15-20 times the tensile strength.

Hoek and Bieniawski (1965) provide a comprehensive discussion of the process of rock failure in compression on the laboratory scale.

**Figure 3** (from Hoek and Bieniawski, op.cit.) shows the results of tests to observe how an initial crack in a glass plate extends when subject to increasing compressive loading.

Note that

1. Increase of the major (compressive) stress  $\sigma_1$  [i.e. a decrease of the ratio  $\sigma_1/\sigma_3$ ] is required to cause the crack to extend;
2. The crack changes orientation as it extends – turning progressively towards the direction of  $\sigma_1$  (i.e. vertical in **Figure 3**) the most stable orientation of a crack under this loading condition.
3. Eventually (in the limit) the tip of the extended crack will be normal to  $\sigma_3$  - and will not extend further.

This crack extension process occurs essentially at each of the myriad of ‘flaws’<sup>5</sup> (in rock, typically associated with grain boundaries) to form a network of **stable** microfractures oriented essentially parallel to the direction of  $\sigma_1$ . Some other mechanism is required to create an instability that leads to collapse of the rock. In test specimens, this mechanism is usually some type of shearing through the micro-cracked system. This is discussed in detail in Hoek and Bieniawski (1965) [see Footnote 9] A good review of current understanding of fracture in a rock mass is presented by Hoek and Martin (2014).

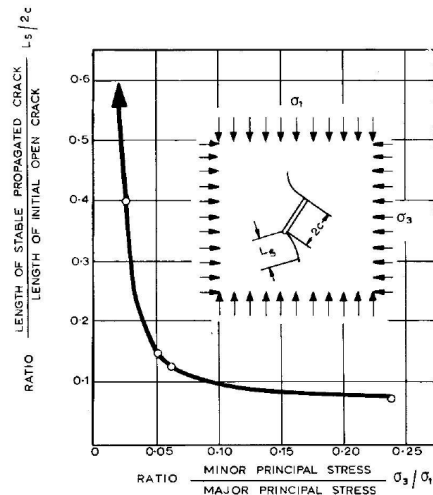


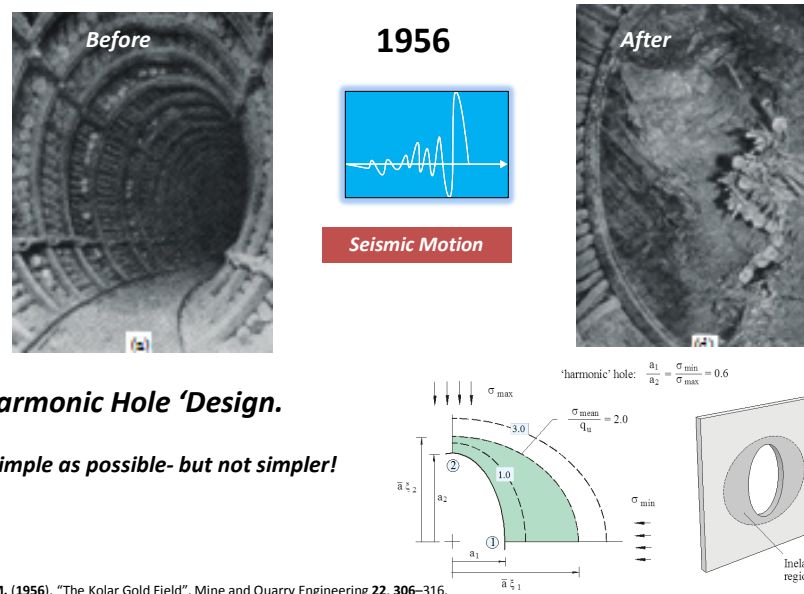
Figure 3. Propagation of a Brittle Crack in Biaxial Compression, Hoek & Bieniawski (1965).

## Design of Underground Mine Openings - using Inglis Solution

In his 1958 book, *Rock Pressure in Mines*, Isaacson (see discussion on page 34) uses the specific case of the Inglis solution shown in **Figure 1(c)** to design the 'optimum shape' for tunnels. It is seen that, for an ellipse with major: minor axis in the same proportion as the major: minor principal stress, the tangential stress will be constant around the periphery. For any other shape of opening, the tangential stress would, at some point around the periphery, be higher than this constant value. Hence, he concluded, this elliptical opening should be the safest for any given depth.

Although superficially plausible, this design makes several assumptions e.g. (i) the rock mass strength is constant around the periphery; (ii) the applied rock loads do not exceed the strength. The design was used in the deep mines of the Kolar Gold Field – where rock bursts were prevalent. In an operating mine, the stress state is constantly adjusting to changes induced by mining. This may result in an instability at a specific extraction location, or these operations may induce slip along a fault in the rock mass. These events produce a seismic wave that will propagate –and perhaps impinge on mine openings remote from the seismic source, as with a classic earthquake. It seems that this occurred in Kolar, with the dramatic consequence shown in **Figure 4**.

### Inappropriate Application of Theory of Elasticity to Mine Opening Design



Ref. Caw, J. M. (1956). "The Kolar Gold Field". *Mine and Quarry Engineering* **22**, 306–316.

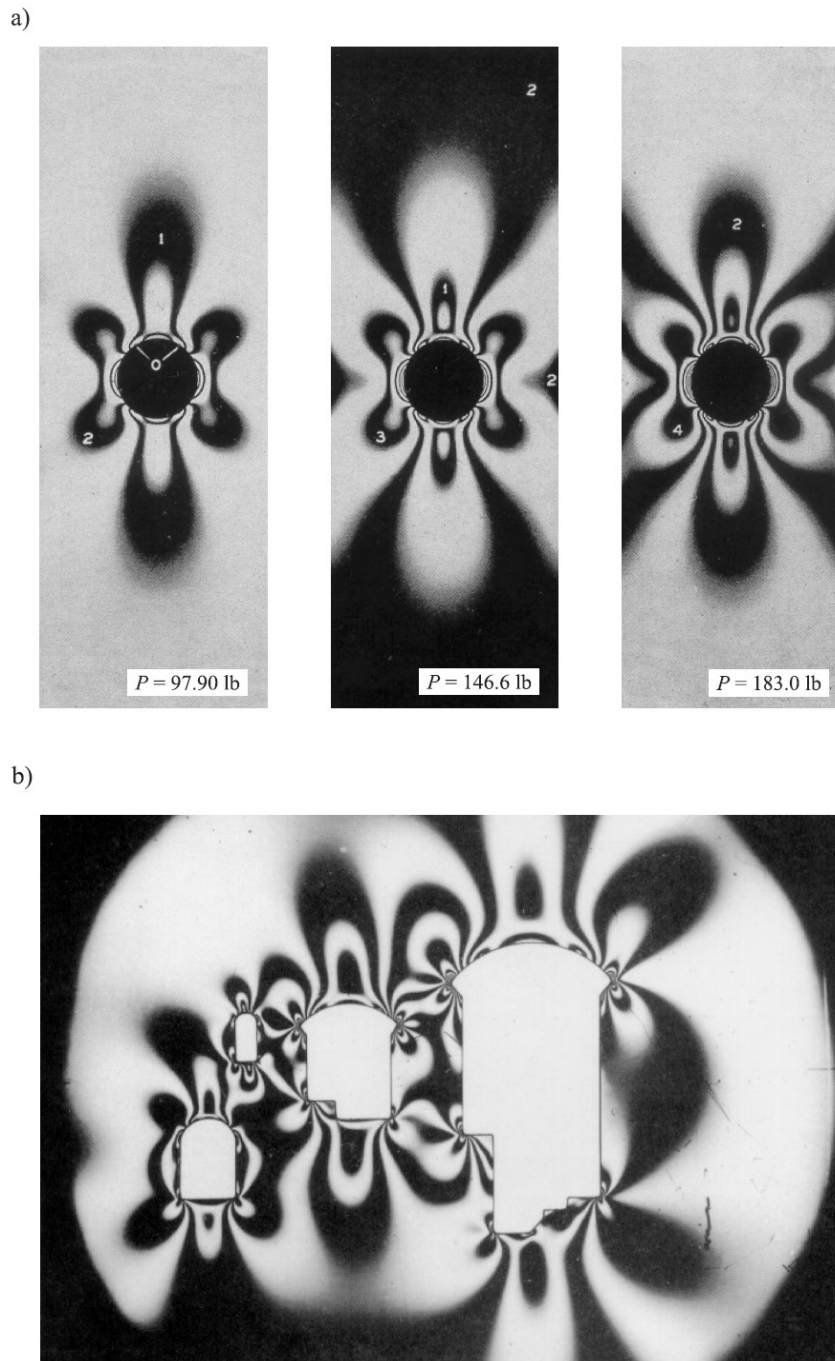
Figure 4. Collapse of Elliptical Tunnels – Gold Mine, Kolar Gold Field. Caw (1956).

The simple elasto-plastic analysis shown in the lower right of **Figure 5**, Fairhurst and Carranza-Torres (2002), serves to illustrate some consequences of over-estimation of the strength of the rock. It is seen that the inelastic region extends a considerable distance into the rock behind the sides of the tunnel but a much shorter distance into the roof and floor. This shape of the failure zone can also be inferred from the elastic solutions of **Figure 1**. As noted earlier, the stress changes around a two-dimensional hole (e.g. Lamé cylinder; Kirsch, or Inglis solutions) decline essentially as the square of the radius of curvature. If we consider the elliptical excavation in terms of the locally variable radius of curvature, we see that the changing extent of inelasticity is consistent with a large radius tunnel on the sides, changing towards a small radius tunnel in the roof and floor. Elliptical tunnels –termed 'harmonic holes'– have sometimes been advocated more recently; but the writer would advise against them.



## Elastic Stress Distribution around Complex Opening Shapes

Although the analytical solutions of **Figure 1** gave a major stimulus to applied mechanics, many of the 'holes' of concern to engineers are of more complex form. Experimental techniques such as photoelasticity were developed to supplement the classical solutions, by providing approximate solutions to 'holes' with irregular boundaries. Techniques allowed both 2D and 3D holes to be analyzed.



**Figure 5.** Photoelastic stress patterns obtained from (a) uniaxial tension of a plate containing a circular hole, Frocht, (1941); model of excavations for a power house, Lang (1962).

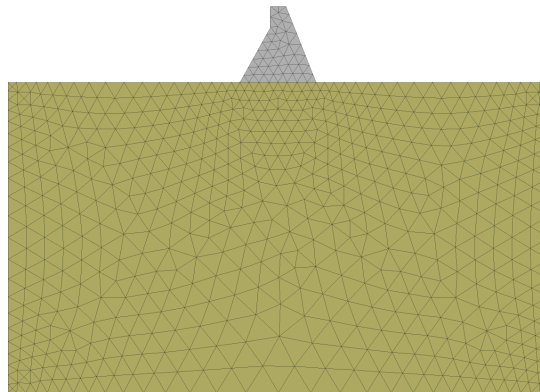
**Figure 5a** shows a photoelastic analysis of stresses around a circular hole. The alternate black and white bands represent stress contours, increasing with proximity to the hole boundary –and giving results that are close to the Kirsch solution for the condition of uniaxial loading. **Figure 5b** shows a similar analysis for the case of excavations for an underground hydroelectric power house –a situation for which there is no analytical solution. It is seen that the stress contours are concentrated in the corner regions of the excavation i.e. where the effective 'radius of curvature' of the boundary is small and wider spaced in the long straight sides and roof regions where the 'effective radius of curvature' is larger. This illustrates that the analytical solutions can serve as a rough guide as to how rapidly the stresses change at various locations around the boundary of a non –circular boundary. In the case of the face of a tunnel, for example, the stress concentrations will tend to decrease as the cube of the distance into

the rock mass i.e. approximating the Lamé solution for the stress distribution around a sphere, whereas in the walls away from the excavation front they will tend to decrease as the square of the radial distance from the excavation periphery into the rock mass.

The advent of high speed computers over the past several decades has led to the development and wide application of approximate numerical techniques that allow classical ‘closed-form’ continuum analyses (such as those in **Figure 1**) to be augmented by approximate, quite accurate stress (and deformation) distribution solutions to more complex shapes –and consideration of more complex constitutive behavior of the material. An early development was the Finite Element Method (FEM) of analysis, one of the earliest to be developed and applied widely in all branches of mechanics. The FEM “subdivides a large problem into smaller, simpler parts that are called finite elements. The simple equations that model these finite elements are then assembled into a larger system of equations that models the entire problem.” [https://en.wikipedia.org/wiki/Finite\\_element\\_method](https://en.wikipedia.org/wiki/Finite_element_method)

**Figure 6** shows a typical plot of a grid of Finite Elements, in this case for a two-dimensional numerical (i.e. computer) analysis of the distribution of stress and deformation in a dam and the rock mass foundation due to filling of the dam. The regions of interest in the continuum are divided into a number of connected elements-smaller in the regions where the stresses and deformations are more concentrated and variable.

The Boundary Element Method (BEM) is another popular technique for developing approximate numerical solutions to stress and displacement distributions in the interior of an elastic body. The BEM uses the boundary conditions of the problem to arrive at the stress at any point in the interior of the model domain, rather than dividing the interior of the solid into elements, as with FEM.



**Figure 6.** Example of a Finite Element grid for a two-dimensional continuum analysis of stresses in a rock foundation beneath a dam.

A third category, Finite Difference Methods (FDM), is based on solving the differential equations involved in a continuum analysis by approximating derivatives as finite difference equations. FDM's are thus classed as discretization methods. A useful brief discussion of the BEM, FEM, FDM methods [plus the Distinct (or Discrete) Element Method (DEM) –to be considered later in this paper], and combinations of the methods to improve computational efficiency of solving problems, is presented in Brady and Brown (2004).

## Consideration of Discontinuities in a Rock Mass

In his opening Presidential address to the First Congress of the International Society for Rock Mechanics (ISRM) in Lisbon 1966, Founder President Leopold Müller stated:

*“Many experts agree with me that discontinuity and anisotropy are the most characteristic properties of the material rock and that the properties of jointed media depend much more upon the joints of the unit rock block system than upon the rock material. Therefore, any theoretical investigation of that material has to go its own ways, in the same way as the construction material of soil years ago suggested to soil mechanics its own methods, which differ greatly from the way of thinking of technical (continuum) mechanics.”*

Although early efforts to develop computational (i.e. numerical) approaches to mechanics were underway in the 1960's, their application to the complex issues of deformation and failure of jointed and fractured rock masses was essentially undeveloped.

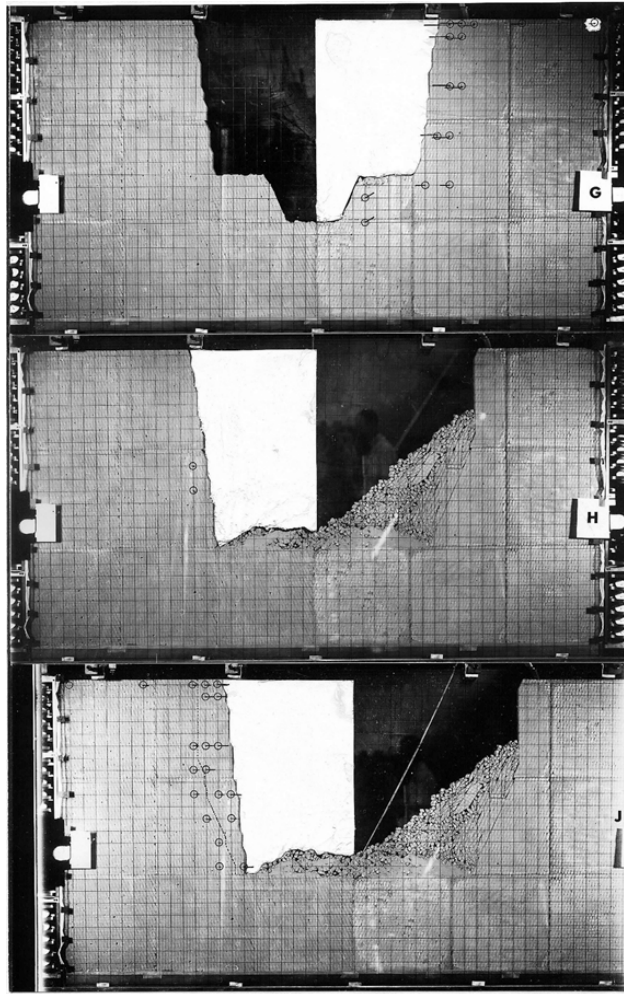
Since numerical modeling techniques for analysis of rock masses containing three-dimensional systems of fractures, faults, bedding planes, and other discontinuities were not generally available, considerable emphasis was placed on experiments using physical models. (The ‘rock’ blocks were usually fabricated from material such as Plaster of Paris, sometimes with additives to scale the strength –and perhaps coefficient of friction between blocks such that a block could slide and/or fail under the gravitational load of the experimental slope - to correspond to such possibilities in the actual rock slope.

**Figure 7** shows a two –dimensional model constructed at Imperial College by N. Barton to simulate the deformation and collapse of a rock slope ca 1970. The model is approximately 2.5 m high.

**Figure 8** shows a three-dimensional physical model of the Alqueva arch dam. A fault in the dam foundation is simulated in the model by introduction of a low friction interface (shown in yellow). The inset in the upper right of **Figure 8** shows a detail of the low friction interface.

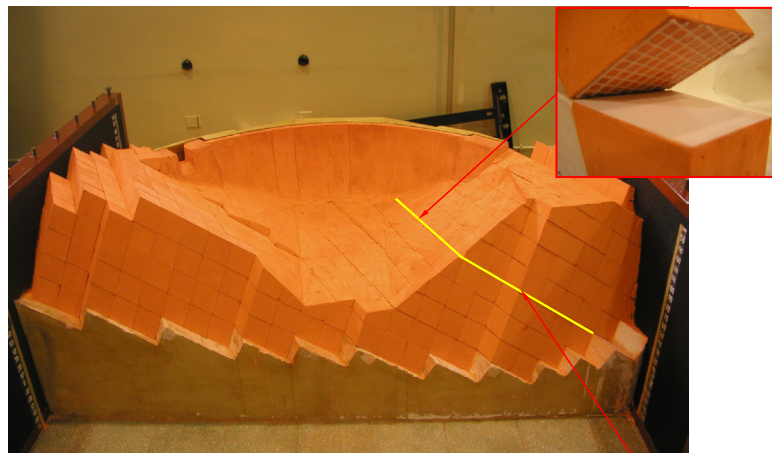
The actual dam is 96 m high. The model scale is 1:250. The overall height of the model is about 1 m, including the foundation.

Such models were used widely internationally, especially for dam foundation design. Although some are still constructed, their use is declining.



*Figure 7. Two-dimensional physical model to simulate excavation of a rock slope in a jointed rock mass.*  
[Courtesy, Dr N, (Nick) Barton.]

***Physical model of Alqueva arch dam***  
(Courtesy J.Lemos LNEC (Lisbon )



***Model with blocky foundation and lower friction fault***

**Figure 8.** Three-dimensional physical discontinuum model of the foundation of an arch dam.  
[constructed at LNEC (National Civil Engineering Laboratory) Lisbon, Portugal.



## Centrifuge Modeling of Geotechnical Structures

Physical models are also used in conjunction with centrifuges to simulate the deformation and failure of rock structures. The deformation and strength properties of the model material are scaled down and the effective gravitational forces increased, by centrifugal acceleration, in order to develop failure of the centrifuged model that is analogous to that of the behavior of the full scale structure.

### The Discrete Element Method (DEM)

Although much of the research on the mechanics of deformation of jointed rock in the 1960's and 1970's emphasized physical models, Peter Cundall, then a graduate student at Imperial College, decided to pursue the possibility of developing a computer model of the interaction of an assembly of rigid blocks at their interfaces. In 1971, he presented the landmark paper, Cundall (1971) at an ISRM Symposium in Nancy, France. Although the method was computationally intensive and computers were primitive compared to the powerful systems now available, this work represented a significant advance towards the goal of modeling a rock mass in terms of the interactions of deformable joints at the block interfaces. [As noted earlier, the procedure, which is termed the Discrete (or Distinct) Element Method (DEM), is discussed briefly in Brady and Brown (Third Edition, 2004) section 6.7 pp189-192] The DEM is intrinsically computationally intensive, but this difficulty is now receding quickly with the advent of massively parallelized and Cloud systems of computation. Several of the US National Laboratories, equipped with very powerful, 'state of the art' computer systems now use the DEM in their studies of geomechanics problems. **Figure 9** illustrates the advances made with DEM modeling of a three-dimensionally jointed rock mass (The Synthetic Rock Mass).

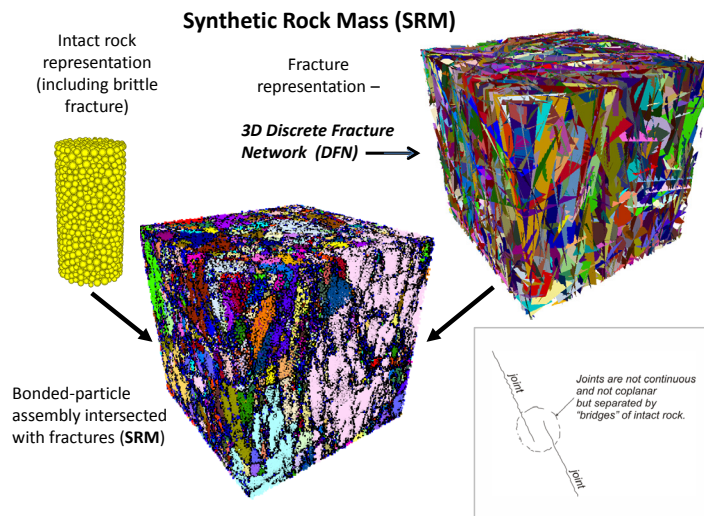


Figure 9. Synthetic Rock Mass (SRM).

A computer model of a deformable rock mass, with deformability and strength properties as determined from tests on intact cores (yellow cylinder in **Figure 9**) is created as an intact cube of rock. A similar cube containing a three-dimensional discrete fracture network [DFN] (frictional, with normal and shear stiffness) of finite (and varied) length, is superimposed on the cube of intact rock. The finite length of fractures creates regions of intact rock within the specimen-onto which the loads applied to the boundaries of the cube are concentrated. It is natural, therefore, that this SRM will exhibit a lower strength than the intact core. Further details of the design and behavior of an SRM are discussed in Mas Ivars et al; (2011).

## Hydraulic Fracturing

### Models based on Inglis Solution

Hydraulic fracturing concerns fractures that are internally pressurized by a fluid. The two-dimensional solution for the stress distribution in the elastic solid for an elliptical hydraulic fracture subject to uniform internal pressure was developed by Sneddon and Green (1946) - and has been the basis for numerous models of the hydraulic fracturing process

Two such models that have been used extensively in the petroleum industry are known by the acronyms PKN (Perkins, Kern and Nordgren) and KGD (Kristianovic, Geertsma, de Klerk)<sup>6</sup> See **Figure 6 (i)**.

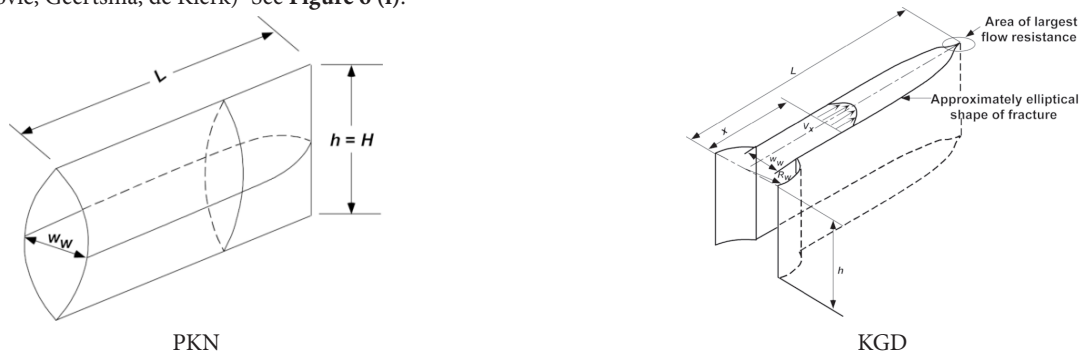


Figure 10. Classical 2D Models for Hydraulic Fracture.



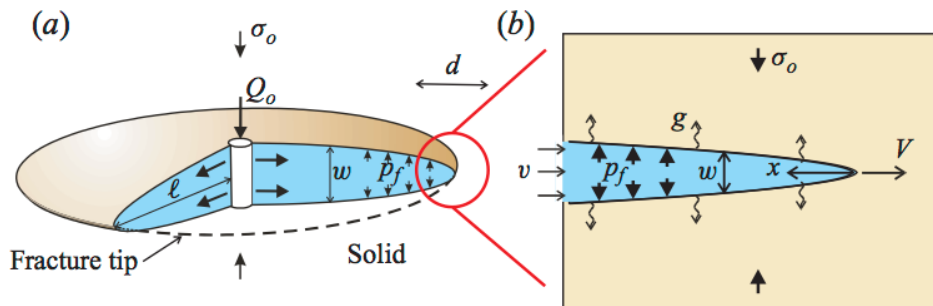
However, these models have serious limitations with respect to representing hydraulic fractures such as are generated in field operations. They do not really address the mechanics of fractures that are propagating driven by pressurized flowing fluids, as is the case in practice.

Barenblatt (1961/62) and his colleagues carried out pioneering research on this problem. These and other studies established that the crack in the region of the tip would propagate ahead of the fluid i.e. there is an ‘unwetted region’ at the crack tip.

### Intermediate Asymptotic Analysis

Detournay and his students have conducted intensive research, over the past two decades, on the mechanics of hydraulic fracture propagation in permeable rock, using the technique of scaling.

The situation for a crack assumed to be extending axisymmetrically at a velocity  $V$  from the borehole, driven by a viscous fluid in a permeable rock (i.e. subject to ‘leak-off’) of a given ‘fracture toughness’ is illustrated in **Figure 1**.



**Figure 11.** Radial Flow Model of a Penny – Shaped Hydraulic Fracture (considers Crack Propagation with Fluid Flow and Leak-off).

Although this combination of rock properties and fracture propagation processes is involved in hydraulic fracturing, there is no ‘exact solution’ to describe the behavior of the propagating crack. Scaling analysis provides valuable insights to be gained on how fracture toughness, fluid viscosity and ‘leak-off’ (rock permeability) assume greater or lesser importance over the entire range of practical situations. In some cases, it is possible, for example, by focusing on the primary energy dissipation mechanism to observe

*“It is possible to utilize a ratio of energy required to extend the fracture to the energy dissipated through viscous fluid flow within the fracture. If this ratio is small, then the fracture is within the viscosity-dominated regime. If this ratio is large then it is considered toughness-dominated.” [Savitski and Detournay (2002)]*

The method is attractive in that, as with the classical closed form analytical solutions discussed earlier (**Figure 1**), the key variables are expressed in dimensionless groups, allowing effects of scale to be readily assessed. Detournay, E, A.P. Peirce and A.P. Bunger (2007) and Detournay (2016) discuss the method, together with some examples of application. The technique is described well in the book, *Scaling* by Barenblatt (2003), which covers also the basic concepts of dimensional analysis and physical similarity that are relevant to this analysis.

Although hydraulic fracturing has been focused, to date, on applications to petroleum engineering, the technique is assuming growing importance in a number of subsurface engineering applications –Enhanced (or Engineered) Geothermal Systems (EGS); CO<sub>2</sub> sequestration; waste-water injection; radioactive waste isolation; ‘pre-conditioning’ of rock in caving mining; ‘borehole mining’– i.e. any application where fluid is injected under pressure into rock and where fractures may be initiated, either deliberately or inadvertently.

## Discussion

The above presentation has attempted to illustrate how, over the years, engineers have gained insight into the mechanics underlying the design of structures in rock. Civil and mining engineers have been involved with tunnels, slopes and other complex excavations for thousands of years. Geological engineers, more focused on the ‘geological setting’ of a project, often work closely with their civil and mining colleagues. Petroleum engineering has a more recent history, starting with the drilling of the 70 ft deep Drake well in Pennsylvania in 1859.<sup>7</sup>

A major distinction is that petroleum engineers must operate their projects remotely, i.e. without direct human access to the subsurface. This has stimulated a strong R&D activity in the petroleum industry, resulting in major technological innovation, of which directional, long reach drilling is a prime example. Civil and mining engineers, with direct, extensive, access to the underground - and much longer experience, have tended to be guided by empirical rules that have evolved over long time. It is important to note the cardinal rule of empiricism – they should not be applied beyond the bounds of the cases from which they have been derived.

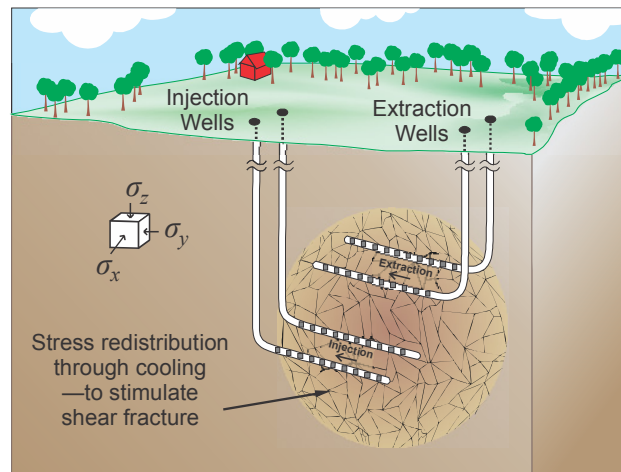
Even rules that have been adopted widely need to be examined critically. The cautionary note re the Hoek-Brown criterion is an example

*“The fact that the criterion works, more by good fortune than because of its inherent scientific merits, is no excuse for the current lack of effort or even apparent desire to find a better way. It is my hope that this short note may catch the eye of someone who has the skill and the motivation to pick up the challenge and to lead in the development of better tools for providing us with the input data which we need for rock engineering designs of the future” E.Hoek. (1994).*

Many projects today are facing conditions for which there is little or no precedent. **Figure 12** shows a hypothetical design for an EGS, intended to extract heat from a large volume of crystalline rock. It is assumed that directional drilling developed for (softer) sedimentary formations will be applicable for these very different conditions. The dynamics of the directional drilling system, the coupled THMC (thermo-hydro-mechanical-chemical) processes that are operative in EGS systems, are not well understood; will the system provoke damaging levels of induced micro-seismicity? The stated energy potential provides a huge incentive to address these issues.

"We estimate the extractable portion [of Geothermal Energy] to exceed 200,000 EJ\* or about 2,000 times the annual consumption of primary energy [in US]..."

Tester, J.F. et al; 2006 "The Future of Geothermal Energy. MIT Synopsis - p. 5(5)



Development of EGS (Enhanced Geothermal Energy) will Require Considerable Technological Innovation.

**Figure 12.** Hypothetical Design of an EGS Project.

Although there have now been several field efforts to develop EGS, each seems to provide surprises. Commenting on what was learned from the Fenton Hill, New Mexico EGS tests (ca 1970) by scientists and engineers at Los Alamos National Laboratory Duchane and Brown (2002) note:

*"The idea that hydraulic pressure causes competent rock to rupture and create a disc-shaped fracture was refuted by the seismic evidence. Instead, it came to be understood that hydraulic stimulation leads to the opening of existing natural joints that have been sealed by secondary mineralization. Over the years additional evidence has been generated to show that the joints oriented roughly orthogonal to the direction of the least principal stress open first, but that as the hydraulic pressure is increased, additional joints open."* (p.15 –last paragraph).

In the ISRM News Journal note by Dr Hoek, referred to earlier [E.Hoek (1994)] he also expresses concern over the lack of effort to obtain field data on rock mass properties

*"I see almost no research effort being devoted to the generation of the basic input data which we need for our faster and better models and our improved design techniques. These tools are rapidly reaching the point of being severely data limited ...."*

While agreeing fully with the view that computer modeling needs to be focused firmly on the specific field problem, the writer believes that the procedure proposed by Starfield and Cundall(1988) provides a practical approach to defining what specific field data can best serve to improve the engineering design

'Numerical experiments', in which numerical analyses are used to examine the possible significance of practical features not considered in classical analyses, can provide valuable insights –helping to take the place of classical laboratory experiments in cases where these are not feasible, as in much of rock engineering.

The paper by Starfield and Cundall (1988) provides valuable insights, guidelines and advice on the effective use of computer models in rock engineering. It is hoped that the following excerpts will encourage the reader to study the complete paper (nine pages) in its entirety. ....*rock mechanics models should never be run only once; it is in the sensitivity of the results to changes in parameters and assumptions that the model is most informing.* The modeling approach that the authors propose *"exemplifies an underlying philosophy. People commonly say you only get out of a computer what you put into it" or "the results are only as good as the data."* At one level these comments are of course true, but at another they are misleading. *Modelling in a cautious and considerate way leads to new knowledge or, at the least, fresh understanding. Even the writer of a modelling program learns new things when running his program; a system of interacting parts often behaves in ways that are surprising to those who specified the rules of interaction. Exploring and explaining these interactions is a form of learning; the model becomes a laboratory for those who built it."* Used properly *"a model is an aid to thought rather than a substitute for thinking"*.

It is recommended also that the model findings be presented to and discussed with colleagues on the project with a variety of relevant backgrounds and experience.

It is useful also to have two (or more) modeling groups analyze the design problem - working independently of each other –and present their findings to the contactor at the same meeting. The discussion both with the design team and between the modelers can be very informative to all. It is not necessary to be a skilled modeler to recognize when a model prediction is contrary to one's own experience. ....*"the most powerful computer in the world isn't nearly as intuitive as the one we're born with."* President Obama<sup>8</sup>.

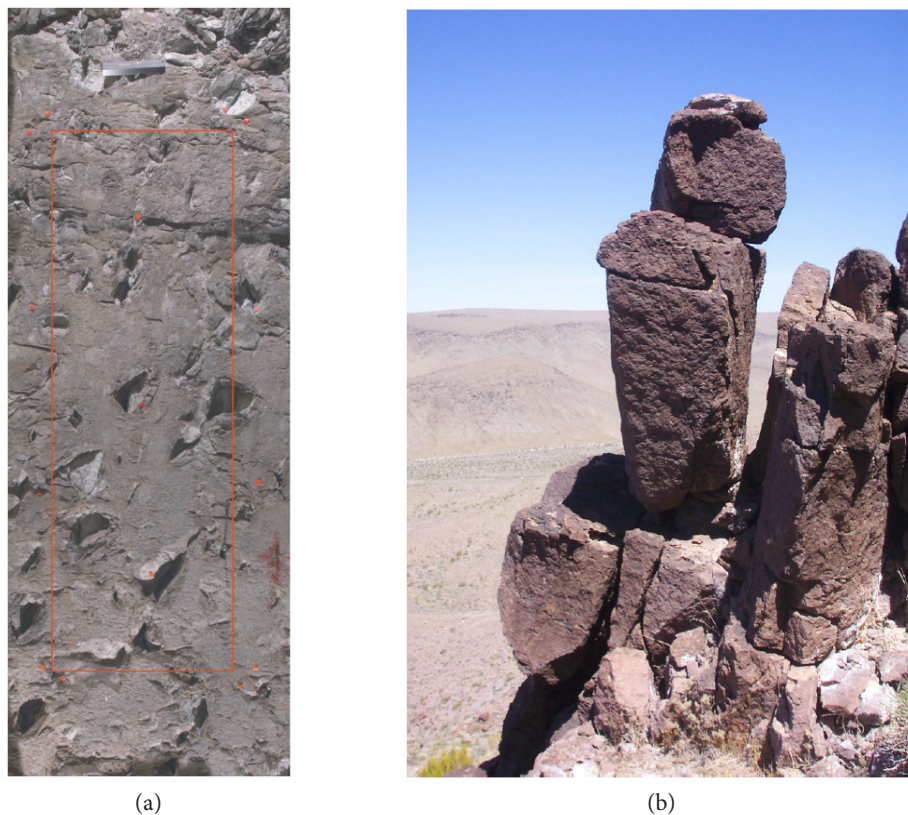
Pells (2005) quotes ISRM President Müller as follows

*“Rock mechanics is one of the scientific disciplines in which progress can only be achieved by means of interdisciplinary team work. ... As a branch of mechanics rock mechanics cannot prosper outside the general fundamentals of the science of mechanics”.*  
Müller (1974)

The following example, part of the studies intended to demonstrate whether or not Yucca Mountain was a suitable site for a high level waste repository in the United States, is an excellent example of multi-disciplinary team work attempting to resolve a difficult issue.

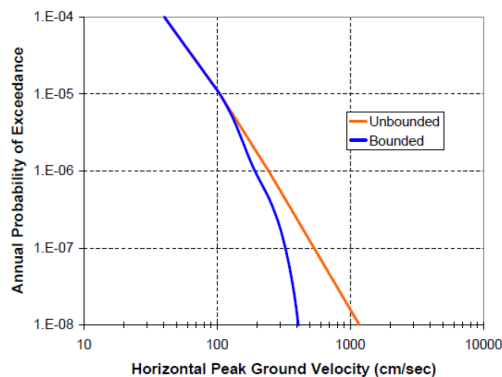
As part of its site characterization studies for Yucca Mountain, the US Department of Energy, was required to establish the extreme ground motions (very high) that could arise at Yucca Mountain at extremely low probabilities of exceedance (hazard). It was required to assess the amplitude of ground motion (peak ground velocity and peak acceleration) over the next 10,000 years (lifetime of the repository) with a probability  $10^{-4}/\text{yr}$  i.e. a probability of  $10^{-8}/\text{yr}$ . A PSHA (Probabilistic Seismic Hazard Assessment) completed in 1998 had arrived at a peak ground velocity (PGV) of 10m/sec. and a peak acceleration of 11g –values significantly higher than ever recorded anywhere. Although almost every seismologist felt that these values were physically unrealizable, they saw no way to constrain them, based simply on statistical extrapolation of known events. The uncertainties involved in such predictions is usually classified to be of two kinds ‘Aleatory uncertainty’ i.e. a measure of the inherent randomness in a set of observations. ‘Epistemic’ uncertainty –a measure of lack of understanding of the physical processes involved.

An interdisciplinary Committee of seismologists, structural dynamics and rock mechanics experts, statisticians, etc. Hanks et al (2013) was established to attempt to find additional physical evidence to reduce the epistemic uncertainty. **Figure 13** illustrates two of the types of evidence ‘uncovered’. **Figure 13(a)** shows a section of a tunnel in the Topopah Springs lithophysal tuff formation at the site. The tuff has an age of 12.8 million years. The walls of the lithophysae (essentially gas pockets in the molten lava that solidified into cavities as the lava cooled.) show growths of delicate slender crystalline strands. The shapes of the lithophysae were mapped carefully to develop a computer model of the rock including the pockets, and the model subject to dynamic loading. This revealed that the cavities would collapse at a Peak Ground Velocity (PGV) of the order of 1m/sec - 2m/sec. Thus, Yucca Mountain had not experienced an earthquake exceeding a PGV of the order of 2m/sec in 12.6 million years. The region around Yucca mountain has a series of ‘Precariously Balanced’ rock structures such as that seen in **Figure 13**. Numerical models of these structures were created and the PGV required to topple them determined. Other geological studies of the structures were able to establish how long they had been in this precariously balanced state –allowing another constraint on the PGV to be developed. The region was also close to the site of underground nuclear tests in the recent past. Several such lines of evidence were compiled, and a revised calculation was undertaken.



**Figure 13.** Underground (a) and Surface (b) Geological Features considered in Assessment of the Earthquake History of Yucca Mountain.

- (a) Photograph from the Enhanced Characterization of the Repository Block tunnel sidewall in the lower unit of the Topopah Springs Tuff, showing traces of lithophysal cavities and vapor phase alteration rims. Size of panel shown is (3m x 1m). Up is to the left. [Hanks et al; (2013) Figure 19, p.32]
- (b) Photo of the precariously balanced rock called “Tripod”, a marginally stable stack of rocks in the Middle Ledge of the Topopah Spring Tuff, west face of Yucca Mountain. The stack is approximately 1.5m tall. Photograph by TC Hanks [p.33 Hanks et al; (2013) Figure 20, p.33]



**Figure 14.** Estimates of Horizontal Peak Ground Velocity at Yucca Mountain for various Probabilities of Exceedance. (Hanks et al; p104).

**‘Unbounded’ refers to estimate based on statistical extrapolation of known earthquakes only. (historical record)**

**‘Bounded’ refers to estimate based on analysis of other geological estimates.**

It is seen that the unbounded value of 11m/sec is reduced to 4m/sec. Although still probably on the ‘high side’ these values indicated that the site met the stringent statutory requirements.

Estimating the probability of extreme earthquakes over this period is unprecedented in seismology, but a multidisciplinary team approach was able to provide a scientifically convincing case that the site met this standard.

Another example of successful interdisciplinary (and, in this case international) co-operation on a geomechanics issue was one in which the writer was personally involved . International outrage in 1995-96 over France’s resumption of nuclear testing in the atolls of the South Pacific led to two studies, one by the IAEA Vienna - on the path followed in the South Pacific by radionuclides released through the groundwater into the Ocean; the other study, led by the writer, examined (i) the structural damage to the atolls by the tests; (ii) the change in groundwater flow paths –and possible contamination – due to the explosions. Full details are available in the report International Geomechanical Commission (1999). *Underground Nuclear Testing in French Polynesia; Stability and Hydrology Issues* (1999) <http://conservancy.umn.edu/handle/11299/162862>

Another exceptional and very successful collaborative effort that deserves mention is the book edited by Drs John Read (CSIRO) and Peter Stacey (2009). This 9 year, \$11 million collaborative effort –with all funds provided by a variety of sponsors , has been hailed as a substantial contribution to a major industrial problem.

Distinguished colleagues in related disciplines have also expressed support for interdisciplinary cooperation in addressing subsurface science and engineering problems. Thus, Dr Chris Scholz, at Lamont Doherty Seismological Laboratory, writes - Scholz (2002)

*“It is a consequence of the way in which science is organized that the scientist<sup>9</sup> is trained by discipline, not by topic, and so interdisciplinary subjects such as this one tend to be attacked in a piecemeal fashion from the vantage of the different specialties that find application in studying it. This is disadvantageous because progress is hindered by lack of communication between the different disciplines, misunderstandings can abound, and different, sometimes conflicting schools of thought can flourish in the relative isolation of separate fields. Workers in one field may be ignorant of relevant facts established in another, or, more likely, be unaware of the skein of evidence that weights the conviction of workers in another field. This leads not only to a neglect of some aspects in considering a question, but also to the quoting of results attributed to another field with greater confidence than workers in that field would themselves maintain. It is not enough to be aware, secondhand, of the contributions of another field - one must know the basis, within the internal structure of the evidence and tools of the field, upon which that result is maintained. Only then is one in a position to take the results of all the disciplines and place them, with their proper weight, in the correct position of the overall jigsaw puzzle.”*

Echoing essentially the same sentiment Cornet (2015) observes,

*“Today geoengineers and geoscientists dealing with the mechanics of Earth materials must speak the same language.”*



## Concluding Comment

Rock mechanics is a central component of a wide variety of important national and international problems that will increase in severity as a rapidly rising global populations places demands on the Earth's subsurface resources. These problems will require urgent attention –and will yield results most effectively if addressed by multi-disciplinary teams of scientists and engineers. The US has a strong tradition of interdisciplinary research at its major universities, but currently has no such teams involved in addressing rock engineering problems. This is a serious situation which must be corrected promptly.

## References

- Barenblatt G.I. (2003) *Scaling* Cambridge University Press 171 p.
- Barenblatt, G. I. (1961/1962), Mathematical theory of equilibrium cracks in brittle fracture. *J. of Applied Mechanics and Technical Physics*, No. 4, pp. 3-56 (1961) (in Russian); *Advances in Applied Mechanics*, VII, pp. 55-129 (1962) (in English).
- Brady, B. H. G. and Brown, E. T. (2004) *Rock Mechanics for Underground Mining*, Third Edition 2004 (Kluwer, Dordrecht, Netherlands). See Section 6.4 Computational methods of stress analysis. et seq. pp.178-196.
- Brown, E. T. (2011). Fifty Years of the ISRM and Associated Progress in Rock Mechanics [http://australiangeomechanics.org/admin/wp-content/uploads/2015/03/46\\_4\\_1.pdf](http://australiangeomechanics.org/admin/wp-content/uploads/2015/03/46_4_1.pdf)
- Brune J. (Editor) (2010) *Extracting the Science (2010): A Century of Mining Research*, Society for Mining, Metallurgy, and Exploration, Inc. Denver 544p. ISBN 978-0-87335-322-9, pp. 156-171. For Table of Contents, see [https://app.knovel.com/web/toc.v/cid:kpESACMR01/viewerType:toc/root\\_slug:extracting-science-century](https://app.knovel.com/web/toc.v/cid:kpESACMR01/viewerType:toc/root_slug:extracting-science-century) ) [Celebrating the 100th anniversary of the founding of the US Bureau of Mines. (1910). Closed by Congress 1995 [[https://en.wikipedia.org/wiki/United\\_States\\_Bureau\\_of\\_Mines](https://en.wikipedia.org/wiki/United_States_Bureau_of_Mines) ]
- Caw, J. M. (1956), The Kolar Gold Field. *Mine and Quarry Engineering* 22(8) 306 – 316
- Cherry, J.T. (1967). Computer calculations of explosion-produced craters. *Int'l J. Rock Mech. Min.Sci. & Geomech. Abstracts*, Vol. 4, 1. Jan.1967, pp 1- 12.
- Cook N.G.W. (1965) The Seismic Location of Rockbursts. *Proc. 5th Symp. Rock Mechanics*, Univ. Minnesota. Pergamon Press (Oxford). Pp. 493-516
- Cook, N.G.W., Hoek, E., Pretorius, J.P.G., Ortlepp, W.D. and Salamon, M.D.G. (1966) *Rock Mechanics Applied to the Study of Rockbursts*. *J. S. Afr. Inst. Min. Metall.* 66: 435-528.
- Cornet, F. H. (2015). *Elements of Crustal Geomechanics*. Cambridge University Press
- Crouch, S. L., Starfield, A. M. (1983) *Boundary Element Methods in Solid Mechanics: With Applications in Rock Mechanics and Geological Engineering*. George Allen & Unwin, London. Also [https://en.wikipedia.org/wiki/Boundary\\_element\\_method](https://en.wikipedia.org/wiki/Boundary_element_method)
- Cundall, P. A. (1971) "A Computer Model for Simulating Progressive Large Scale Movements in Blocky Rock Systems," in *Proceedings of the Symposium of the International Society for Rock Mechanics* (Nancy, France, 1971), Vol. 1, Paper No. II-8. See also Cundall, P. A. "Discussion in Symposium on Rock Fracture," in *Proceedings of the Symposium of the International Society for Rock Mechanics*, Vol. 2, pp. 129-132.
- Detournay, E. (2016) *Mechanics of Hydraulic Fractures*. *Annual Review of Fluid Mechanics* Vol. 48: 311-339 (Volume publication date January 2016) DOI: 10.1146/annurev-fluid-010814-014736
- Detournay, E., A.P. Peirce, A.P. Bunger (2007). Viscosity dominated hydraulic fractures. In: E. Demands - *Proceedings 1st Canada-U.S. Rock Mechanics Symposium*, Vancouver, Canada, May
- Duchane, D., Brown, D. (2002). Hot Dry Rock (HDR) Geothermal Energy Research and Development at Fenton Hill, New Mexico. *GHC Bulletin*, December 2002, 13 – 10
- Durrheim R.J. (2010) Mitigating the Risk of Rockbursts in the Deep Hard Rock Mines of South Africa: 100 Years of Research. [Chapter 17 in Brune J. (2010) *Extracting the Science* [http://africaarray.psu.edu/publications/pdfs/SME100\\_Durrheim\\_Rockburst%20research.pdf](http://africaarray.psu.edu/publications/pdfs/SME100_Durrheim_Rockburst%20research.pdf)
- Eberhardt, E, D. Stead and T. Morrison, eds. *Rock Mechanics: Meeting Society's Challenges and Demands - Proceedings 1st Canada-U.S. Rock Mechanics Symposium*, Vancouver, Canada, May 27-31, 2007; Taylor & Francis: London, 2 volumes, 1728pp. ISBN 0415444012
- Fairhurst, C., and C.Carranza-Torres. "Closing the Circle — Some Comments on Design Procedures for Tunnel Supports in Rock," *Proceedings of the University of Minnesota 50th Annual Geotechnical Conference* (February 2002), pp.21-84. J. F. Labuz and J. G. Bentler, Eds. Minneapolis: University of Minnesota, 2002.
- Frocht M.M. (1941) *Photoelasticity* Wiley (New York)
- Griffith, A. A. (1921). The phenomenon of Rupture and Flow in Solids *Philosophical transactions of the Royal Society of London*, Series A. Vol 221, 163 – 198
- Griffith, A.A. (1924). Theory of Rupture. *Proc. First Int. Cong. Applied Mech* (Eds Bienzo and Burgers). 55-63. Delft: Technische Boekhandel and Drukkerij.
- Hanks, T.C., Abrahamson, N.A., Baker, J.W., Boore, D.M., Board, M., Brune, J.N., Cornell, C.A., and Whitney, J.W.( 2013), *Extreme ground motions and Yucca Mountain: U.S. Geological Survey Open-File Report 2013–1245*, 105 p., <http://dx.doi.org/10.3133/ofr20131245>.
- Hoek, E. (1994). [https://www.isrm.net/fotos/gca/1332169399\\_isrm\\_newsjournal\\_-\\_1994,\\_volume\\_2,\\_number\\_2\\_web.pdf](https://www.isrm.net/fotos/gca/1332169399_isrm_newsjournal_-_1994,_volume_2,_number_2_web.pdf) See pp.23-24.
- Hoek, E. and Bieniawski, Z. T. (1965) Brittle Rock Fracture Propagation In Rock Under Compression *International Journal of Fracture Mechanics* 1(3), 137-155 1965 <https://www.rocscience.com/documents/hoek/references/H1965B.pdf>
- Hoek, E., Martin, C. D. (2014). Fracture initiation and propagation in intact rock – A review. *Journal of Rock Mechanics and Geotechnical Engineering*, Vol 6, Issue 4, August 2014, 287 - 300
- Inglis, C.E. (1913) Stresses in a plate due to the presence of cracks and sharp corners. *Trans. Inst'n Nav. Architects*. London. Vol. 55, pp. 219-230
- International Geomechanical Commission (1999). *Underground Nuclear Testing in French Polynesia; Stability and Hydrology Issues*. <http://conservancy.umn.edu/handle/11299/162862>
- Isaacson, E. (1958), *Rock Pressure in Mines*, Mining Publications (London)
- Jaeger J.C., Cook, N. G. W. and Zimmerman, R. W. (2007) *Fundamentals of Rock Mechanics*, Fourth Edition, Blackwell. Chapter 8.9 pp.231-251.
- Kirsch G. (1898). Die Theorie der Elastizität und die Bedürfnisse der Festigkeit. *Verlag des Vereins Deutscher Ingenieure* 42, 797–807
- Krishnamurthy, R. and Shringarputale, S.B. (1990) Rockburst Hazards in Kolar Gold Fields, *Proc. 2nd Symp. Rockbursts and Seismicity Mines Balkema* (Rotterdam) pp 411-420.
- Lamé, G. (1852). *Leçon sur la théorie mathématique de l'élasticité des corps solides*. Paris: Gauthier-Villars
- Lang, T.A. (1963) *Notes on Rock Mechanics and Engineering for Rock Construction*, Berkeley 1962-64 Volumes I and II (personal communication.)

- Mas Ivars, D., M. E. Pierce, C. Darcel, J. M. Reyes-Montes, D. O. Potyondy, R. P. Young, and P. A. Cundall. (2011) The synthetic rock mass approach for jointed rock mass modelling, *International Journal of Rock Mechanics & Mining Sciences* 48 219–244 (2011)
- Müller L. (1974). *Rock Mechanics* – Springer – Verlag, 2nd printing
- Pells, P.J.N. (2000) What happened to the Mechanics in Rock Mechanics and the Geology in Engineering Geology? Sixth International Symposium on Ground Support in Mining and Civil Engineering Construction. J. S. Afr. Inst. Min. Metall. vol.108 n.6 Johannesburg Jun. 2008
- Read, J. and P. Stacey (2009) *Guidelines for Open Pit Slope Design* CRC Press 510 p. ISBN 9780415874410 - CAT# K11148
- Ryder J.S. (1988). Excess shear stress in the assessment of geologically hazardous situations J. S. Afr. Inst. Min. Metall., vol. 88, no. 1. Jan. 1988. pp. 27–39
- Salamon, M.D.G. (1974), *Rock Mechanics of Underground Excavations; General Report*. Proc. Third Congress, Int'l. Soc. Rock Mech. Denver. Vol. 1-B, pp.851-1099.
- Savitski, A and E. Detournay (2002), Propagation of a Penny-Shaped Fluid-Driven Fracture in an Impermeable Rock: Asymptotic Solutions, *Int. J. Solids Structures*, Vol 39, No 26, pp. 6311–6337.
- Scholz, C.H. (2002) *The Mechanics of Earthquakes and Faulting* (2nd Edition) Cambridge Univ. Press 471p. Extract from Preface to First Edition.
- Starfield, A. M., Cundall, P. A. (1988). Towards a Methodology for Rock Mechanics Modelling” *International Journal of Rock Mechanics and Mining Sciences & Geomechanics Abstracts*. Vol 25, Issue 3, June 1988, 99 – 106
- Zienkiewicz, O.C. (1968) *Continuum Mechanics as an Approach to Rock Mass Problems*. Chapter 8, pp.237–273. In Stagg, K.G. and O. C. Zienkiewicz, (Eds) *Rock Mechanics in Engineering Practice*. Wiley, New York, 1968. 442 pp.

## Endnotes

<sup>1</sup> With apologies to Offenbach – Orpheus in the Underworld (1858). For many, the term ‘underground’ or ‘underworld’ tends to provoke an instinctively negative reaction. This is not at all new; for ancient Greeks, the underworld was the ‘place for the dead’- the ‘afterlife’. The classic Greek tragedy of Orpheus’ journey to the underworld to bring back his young bride Eurydice, who had just died, builds on this grim image.

Offenbach’s ability to see the beauty within the tragedy –as expressed in his operetta ‘Orpheus in the Underworld,’ invites comparison with the fact that Newton and his laws, though not yet fully revealed, also remain intact and powerful in our ‘Underworld.’

See a remarkable performance by the Kranj Youth Symphony Orchestra, Slovenia  
*Overture (10m 14s)* [https://www.youtube.com/watch?v=vEnW5\\_GTooI](https://www.youtube.com/watch?v=vEnW5_GTooI)  
*Can-Can (2m 04s)* <https://www.youtube.com/watch?v=sf9CtbLGzgw>

<sup>2</sup> “In 1994, the Leon Commission reported that more than 69 000 mineworkers [in South Africa] had lost their lives from 1900 to 1994, while a million had been injured. The biggest contributors to fatalities and injury were rockfalls, rockbursts and seismicity” <http://www.miningweekly.com/article/can-south-africa-stop-the-mine-fatalities-2008-02-01>

<sup>3</sup> The Kolar Gold Field mines were taken over by the Indian government in 1972 and closed in 2001.

<sup>4</sup> “Mathematical analysis in those days was built on the concept of continuous geometrical space in which it was possible to consider individual segments and to introduce the processes of differentiation and integration on this basis. The universally recognized Newtonian molecular theory of structure of bodies, on the other hand, represented them as discrete media composed of individual particles that are connected with each other by the forces of mutual attraction and repulsion” [Filonenko- Boroditch (ca 1960) *Theory of Elasticity* .Preface]

<sup>5</sup> Some cracks may be shielded from the applied stress field by proximity to other (e.g. slightly longer,) cracks.

<sup>6</sup> A discussion of the PKN and KGD models –and others - can be found in [http://petrowiki.org/Fracture\\_propagation\\_models](http://petrowiki.org/Fracture_propagation_models)

<sup>7</sup> [https://en.wikipedia.org/wiki/Drake\\_Well](https://en.wikipedia.org/wiki/Drake_Well)

<sup>9</sup> This is true also in engineering.

## Biography



**Charles Fairhurst**, Professor Emeritus, University of Minnesota, Minneapolis, USA; Senior Consultant, Itasca International Inc. Minneapolis, obtained his Ph.D. in Mining Engineering from the University of Sheffield, UK in 1955. He joined the University of Minnesota faculty, School of Mines and Metallurgy in 1956, serving as Head for several years to 1970, when the Mining program was joined with Civil Engineering to form the Department of Civil and Mineral Engineering. He served as Head of the joint Department from 1973–87, and retired in 1997.

He has consulted on rock stability problems for tunnels, dams, mines and excavations throughout the world. He remains active in

consulting, with a current emphasis on the mechanics of fracture propagation in naturally fractured rock and the effective stimulation of geothermal reservoirs. He served as President of the International Society for Rock Mechanics from 1991–1995, and has been elected to the U.S. National Academy of Engineering (1991) and the Royal Swedish Academy of Engineering Sciences (1979). He is a Fellow of the American Rock Mechanics Association.

Dr. Fairhurst holds honorary doctorate degrees from the University of Nancy, France; St. Petersburg Mining Academy, Russia; University of Sheffield, England; and University of Minnesota, USA; and is Advisory Professor to Tongji University, Shanghai, China.

In December, 2013, he was inducted as Officier, Légion d’Honneur, France.