Overview of Slope Stability Analysis Methods

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Loren Lorig

- Joined Itasca in 1985

- General Manager of Itasca's Chile office (1993-2008)

- CEO (2008-2016)

- Currently Senior Principal Geotechnical Consultant

- Worked as a consultant at most large open pits in worldwide

GEOMECHANICS • HYDROGEOLOGY • MINING • CIVIL • ENERGY

Topics

- Key references

- Physical models

- Analytical methods

- Empirical methods

- Limit equilibrium methods

- Traditional numerical methods

- New numerical methods

- Comparison of limit equilibrium and numerical methods

- Important questions

- Final thoughts

Many other topics important to slope stability analysis are **not** discussed, e.g.:
- *Rock mass characterization*
- *Strength estimation*
- *Hydrogeology*
- *Rockfall/Runout*

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Key References
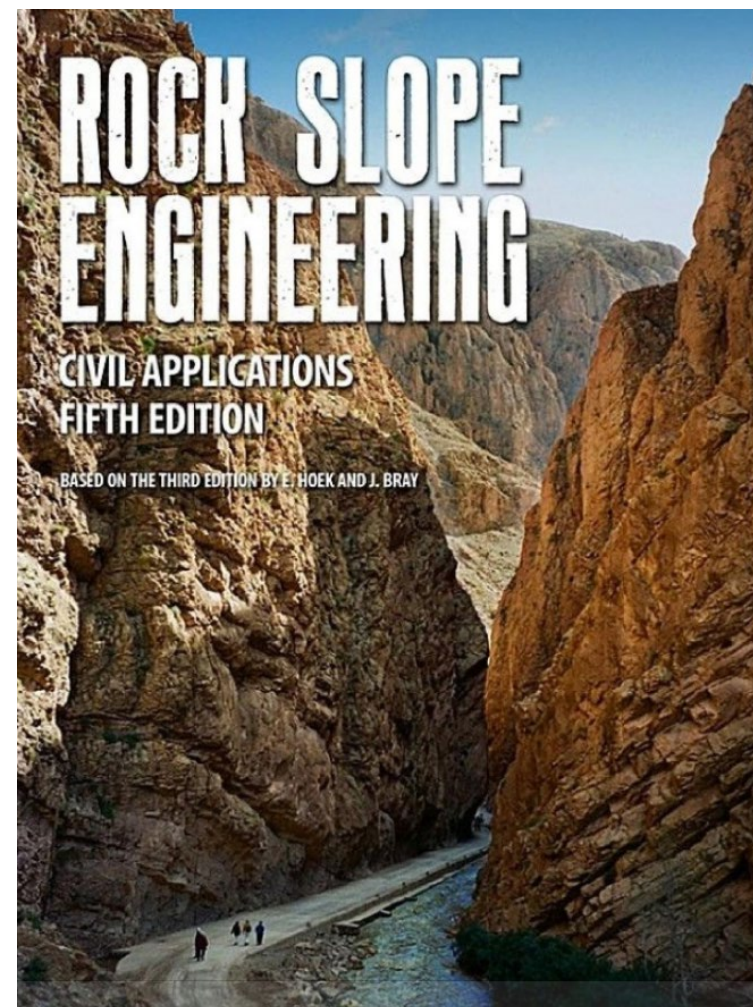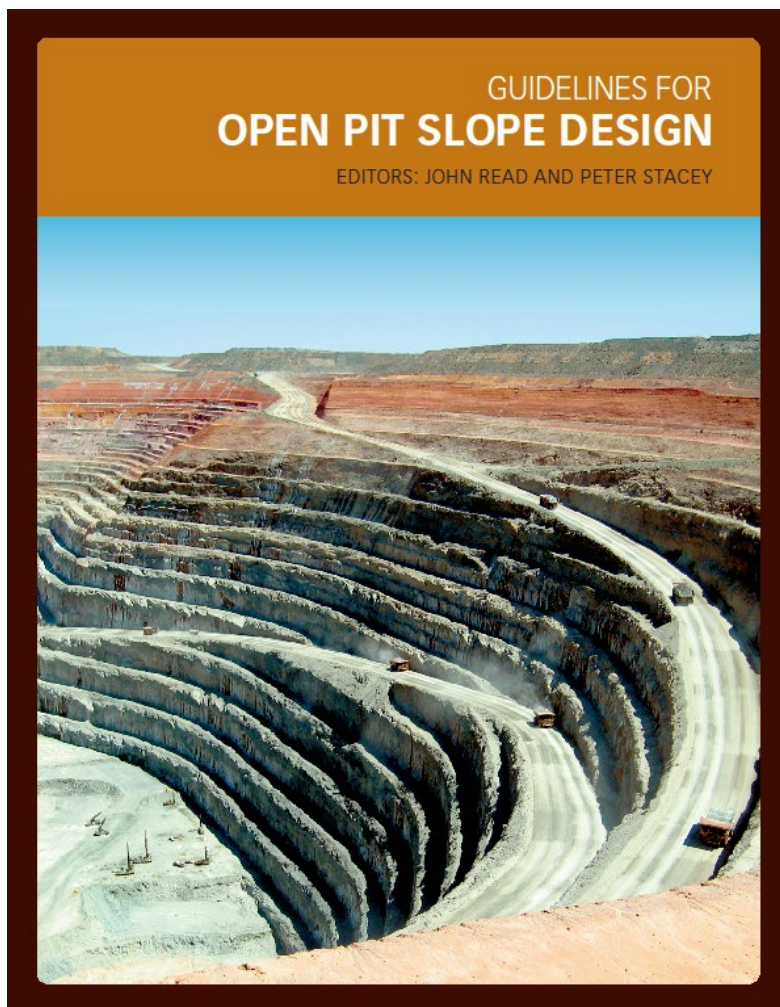
Physical Models

- Used mainly before computers

- Rarely used now

Physical Models

- Base friction model, suggested by Professor Dick Goodman of the University of California at Berkeley, used to simulate simple block movement such as toppling of jointed columns. This model prompted Peter Cundall to write an early version of *UDEC* which was published at a conference in Nancy, France, in 1971.
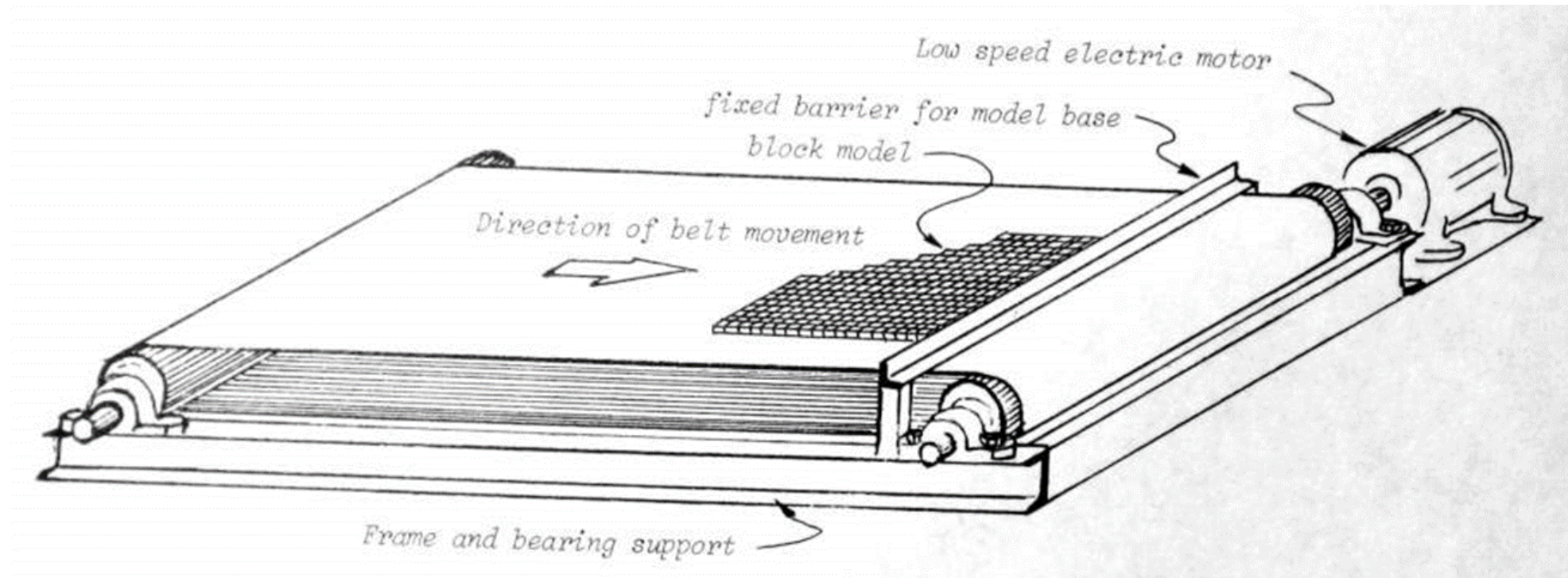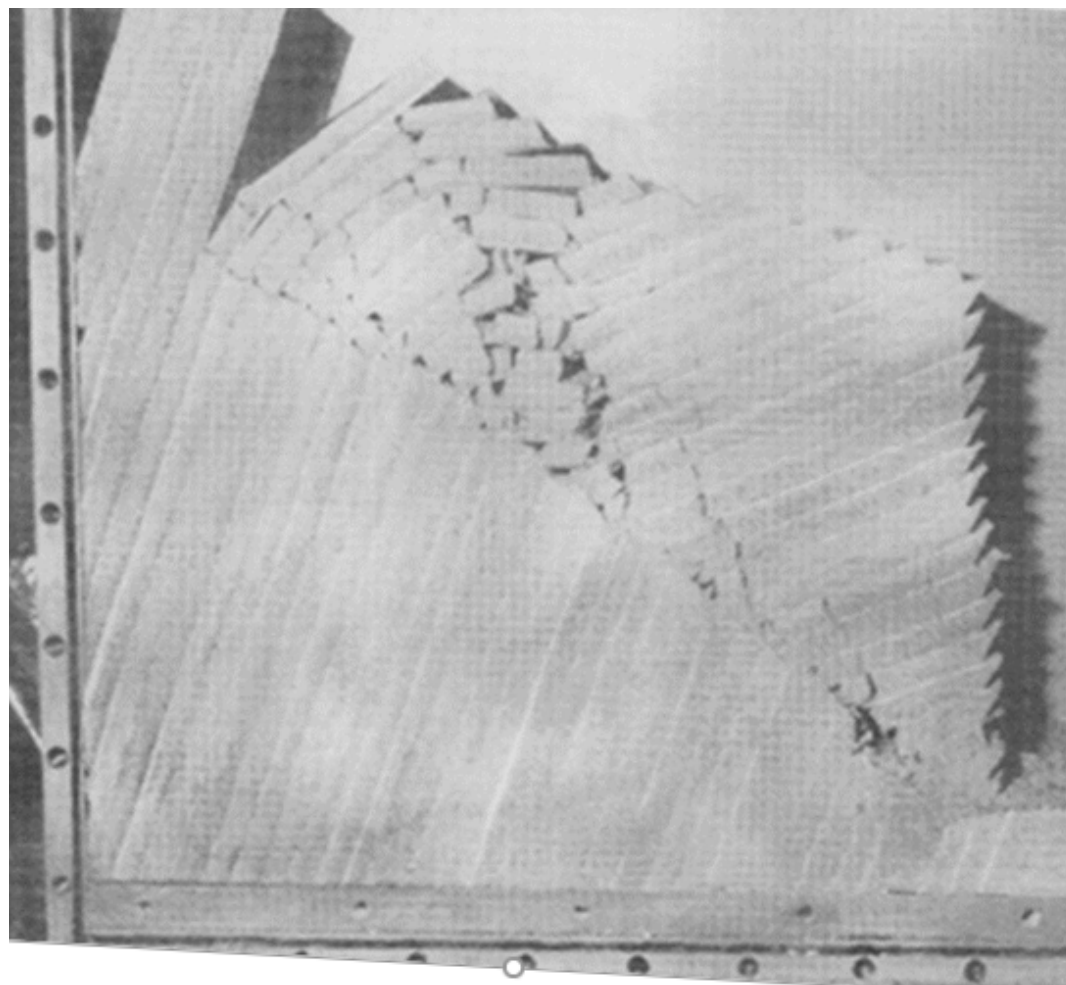
Physical Models

- Jointed rock slope model used by Nick Barton for his PhD at Imperial College, London, in about 1972.
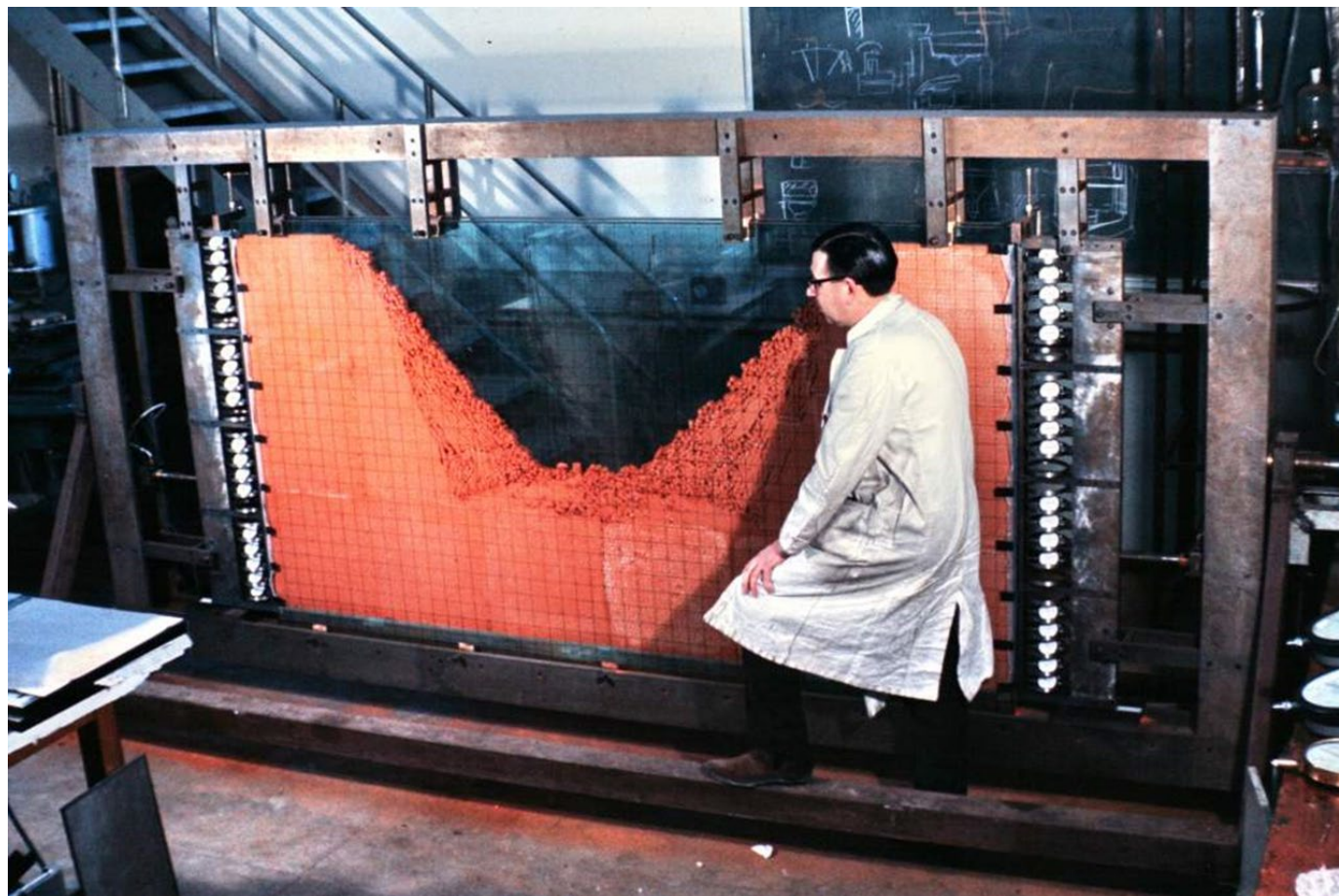
Physical Models

- Centrifuge tests on a synthetic material containing multiple parallel joint planes. Adhikary et al (1997).
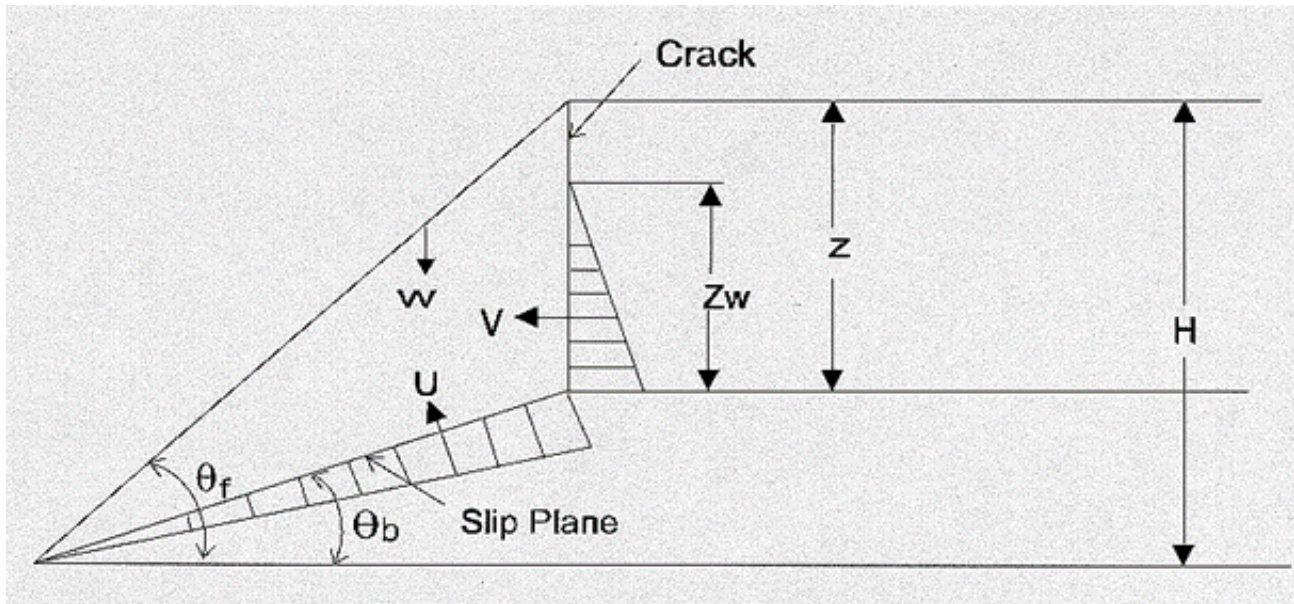
Analytical Methods

- Plane Failure

- Wedge Failure

- Circular Failure
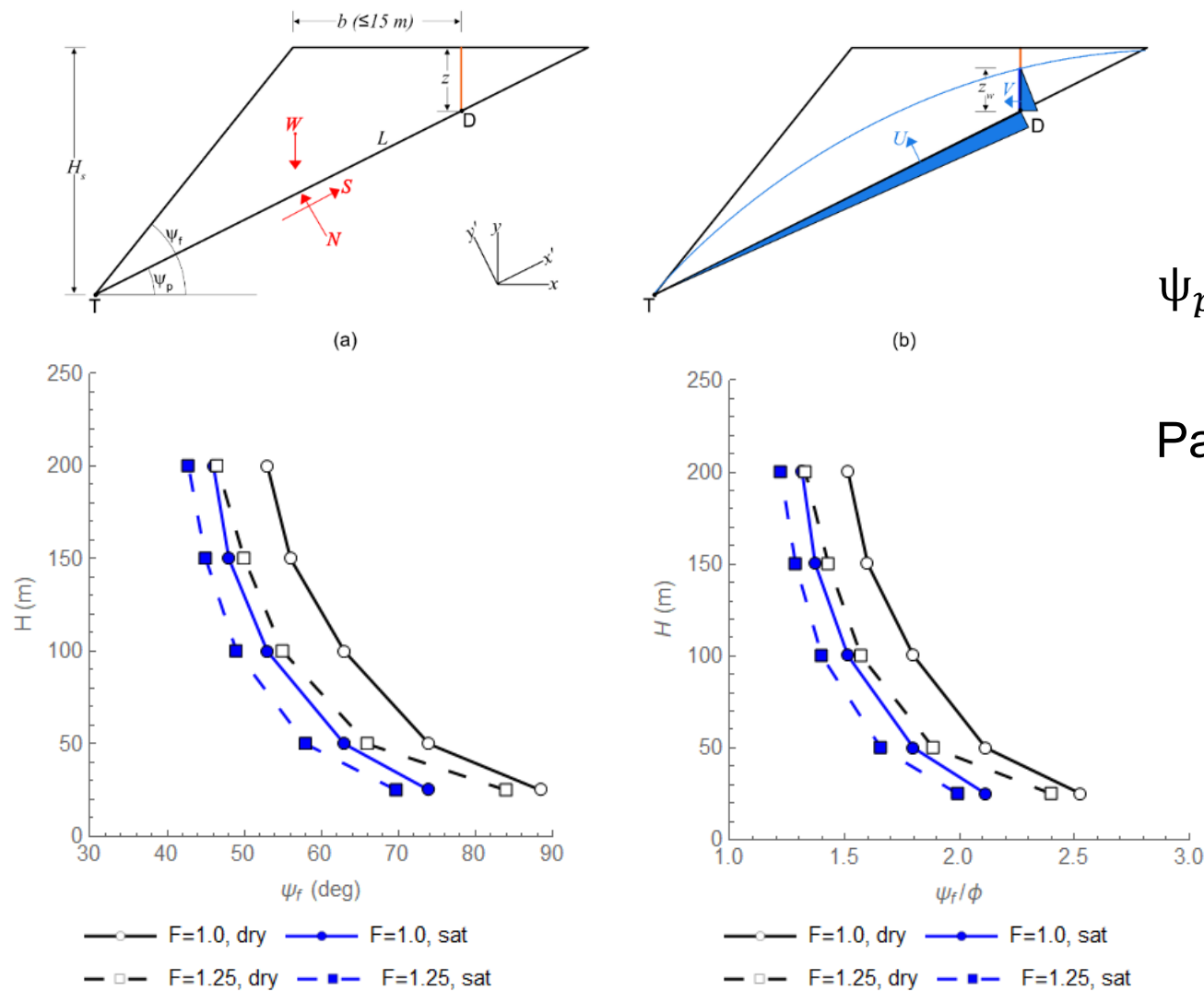
- Toppling Failure

Classic Planar Stability Case

- The planar stability analysis method of sliding of a single block in two dimensions modified from (Wyllie 2004).
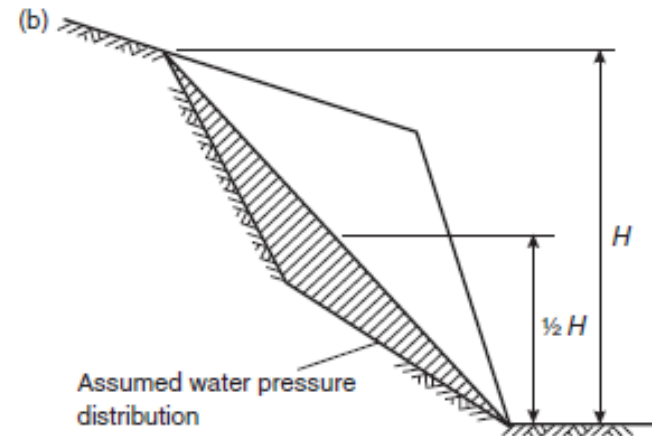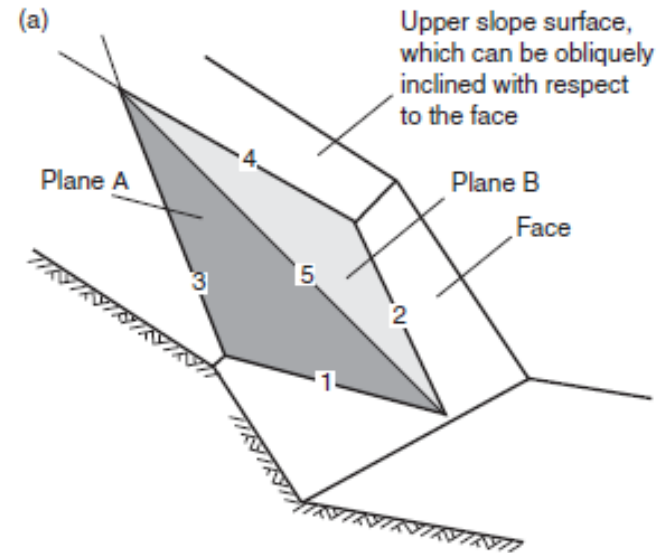

H = Height of Slope
Z = Depth to Intersection of Crack and Failure Plane
Z_w = Height of water in Crack
W = Weight of Rock Mass
U = Force of Water Acting on Daylighted Structure
V = Force of Water Acting on Fault
θ_f = Face Angle of Slope
θ_B = Dip Angle of Daylighted Structure
γ_r = Density of Rock
γ_w = Density of Water
c = Cohesion of Daylighted Structure
ø = Friction Angle of Daylighted Structure
A = Area of Surface of Daylighted Structure

$$FS = \frac{F_{Resisting}}{F_{Driving}} = \frac{cA + \left(W \cos \psi_p - U - V \sin \psi_p\right) \tan \phi}{W \sin \psi_p + V \cos \psi_p}$$

Classic Planar Stability Case

(a)   (b)

$$\psi_p = \frac{1}{2}\left(\psi_f + \phi\right)$$

Parameters c = 110 kPa, ϕ = 35°, γ = 25 kN/m³

Wedge Failure

(a) Plane A, Plane B, Face. Upper slope surface, which can be obliquely inclined with respect to the face.

(b) Assumed water pressure distribution. H, $\frac{1}{2}H$

$$FS = \frac{3}{\gamma_r H}(c_A X + c_B Y) + \left(A - \frac{\gamma_w}{2\gamma_r}X\right)\tan\phi_A$$

$$+ \left(B - \frac{\gamma_w}{2\gamma_r}Y\right)\tan\phi_B \qquad (7.9)$$

$$X = \frac{\sin\theta_{24}}{\sin\theta_{45}\cos\theta_{2.na}}$$

$$Y = \frac{\sin\theta_{13}}{\sin\theta_{35}\cos\theta_{1.nb}}$$

$$A = \frac{\cos\psi_a - \cos\psi_b\cos\theta_{na.nb}}{\sin\psi_5\sin^2\theta_{na.nb}}$$

$$B = \frac{\cos\psi_b - \cos\psi_a\cos\theta_{na.nb}}{\sin\psi_5\sin^2\theta_{na.nb}}$$

40 YEARS

ITASCA

Wedge Stability Calculation Sheet

Input data	Function value	Calculated values

$\psi_a = 45°$ $\cos \psi_a = 0.707$

$\psi_b = 70°$ $\cos \psi_b = 0.342$

$\psi_s = 31.2°$ $\sin \psi_s = 0.518$

$\psi_{na.nb} = 101°$ $\cos \psi_{na.nb} = -0.191$

$\sin \psi_{na.nb} = 0.982$

$$A = \frac{\cos \psi_a - \cos \psi_b \cos \theta_{na.nb}}{\sin \psi_s \sin^2 \theta_{na.nb}} = \frac{0.707 + 0.342 \times 0.191}{0.518 \times 0.964} = 1.548$$

$$B = \frac{\cos \psi_b - \cos \psi_a \cos \theta_{na.nb}}{\sin \psi_s \sin^2 \theta_{na.nb}} = \frac{0.342 + 0.707 \times 0.191}{0.518 \times 0.964} = 0.956$$

$\theta_{24} = 65°$ $\sin \theta_{24} = 0.906$

$\theta_{45} = 25°$ $\sin \theta_{45} = 0.423$

$\theta_{2.na} = 50°$ $\cos \theta_{2.na} = 0.643$

$$X = \frac{\sin \theta_{24}}{\sin \theta_{45} \cos \theta_{2.na}} = \frac{0.906}{0.423 \times 0.643} = 3.336$$

$\theta_{13} = 62°$ $\sin \theta_{13} = 0.883$

$\theta_{35} = 31°$ $\sin \theta_{35} = 0.515$

$\theta_{1.nb} = 60°$ $\cos \theta_{1.nb} = 0.500$

$$Y = \frac{\sin \theta_{13}}{\sin \theta_{35} \cos \theta_{1.nb}} = \frac{0.883}{0.515 \times 0.5} = 3.429$$

$\phi_A = 30°$ $\tan \phi_A = 0.577$

$\phi_B = 20°$ $\tan \phi_B = 0.364$

$\gamma_r = 25 \, \text{kN/m}^3$ $\gamma_w/2\gamma_r = 0.196$

$\gamma_w = 9.81 \, \text{kN/m}^3$ $3c_A/\gamma H = 0.072$

$c_A = 24 \, \text{kPa}$ $3c_B/\gamma H = 0.144$

$c_B = 48 \, \text{kPa}$

$H = 40 \, \text{m}$

$$FS = \frac{3}{\gamma_r H}(c_A X + c_B Y) + \left(A - \frac{\gamma_w}{2\gamma_r} X\right) \tan \phi_A + \left(B - \frac{\gamma_w}{2\gamma_r} Y\right) \tan \phi_B$$

$$FS = 0.241 + 0.494 + 0.893 - 0.376 + 0.348 - 0.244 = 1.36$$

Circular Failure

- From Hoek, E. and J.W. Bray (1981), "Rock Slope Engineering". Revised Third Edition, Institution of Mining and Metallurgy, 358 p.
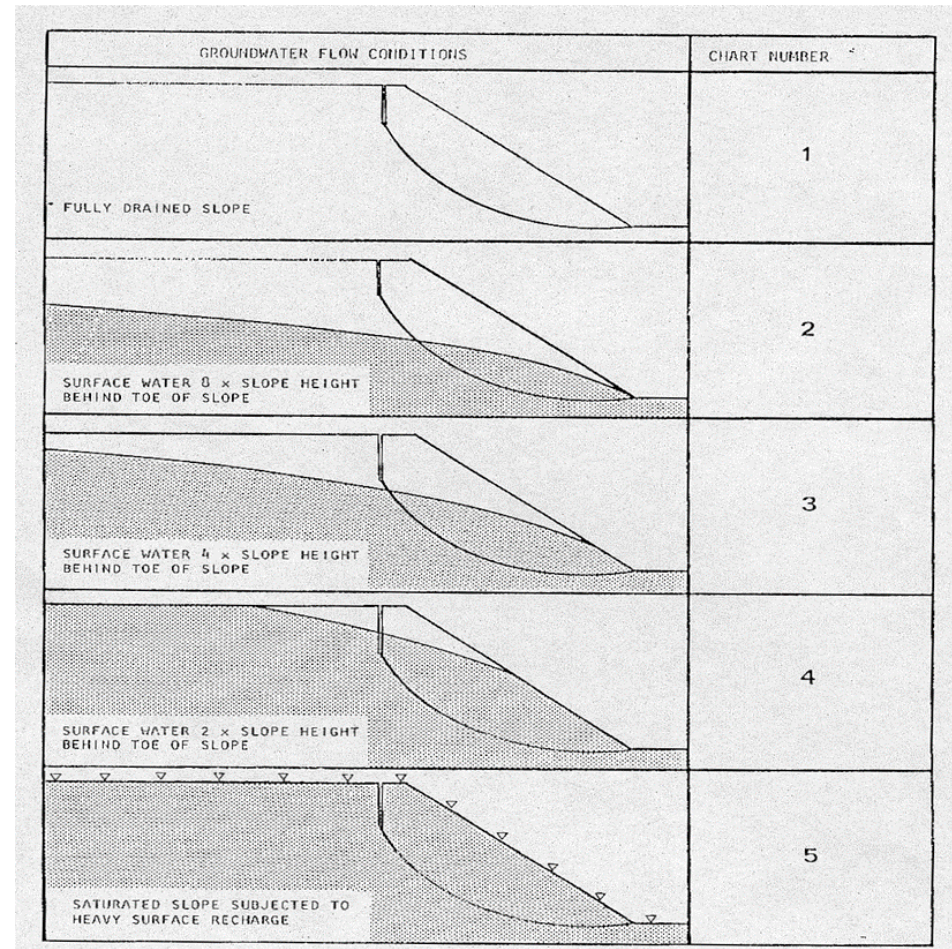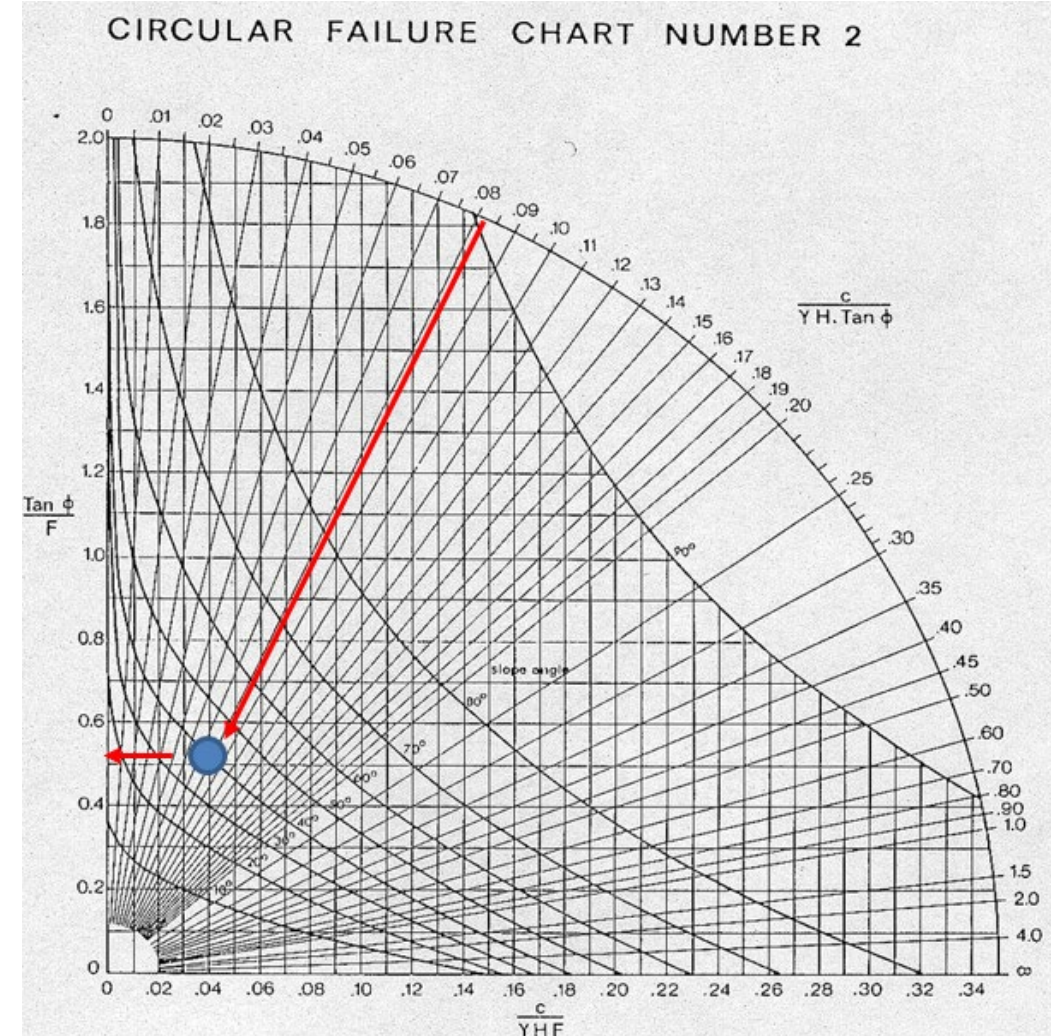
Example: Circular Failure

Given:

Slope Height (H)	= 200 meters
Slope Angle	= 40°
Friction Angle	= 30°
Cohesion	= 0.23 MPa
Density	= 2500 kg/m³

Question:

What is the Factor of Safety, F, for Design Chart 2?

Solution:

$C/\gamma H \tan\phi$	= 0.08
$\tan\phi/F$	= 0.51
F	**= 1.13**

Block Toppling

- Simplified Geometry Analytical Solution by Goodman & Bray (1976)
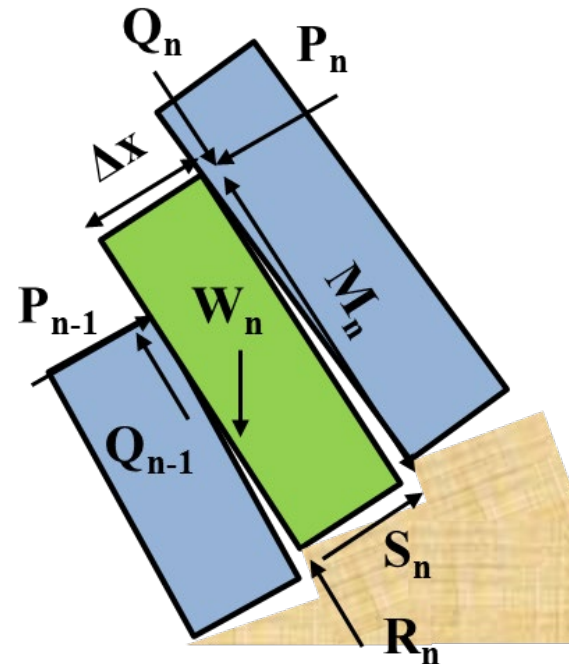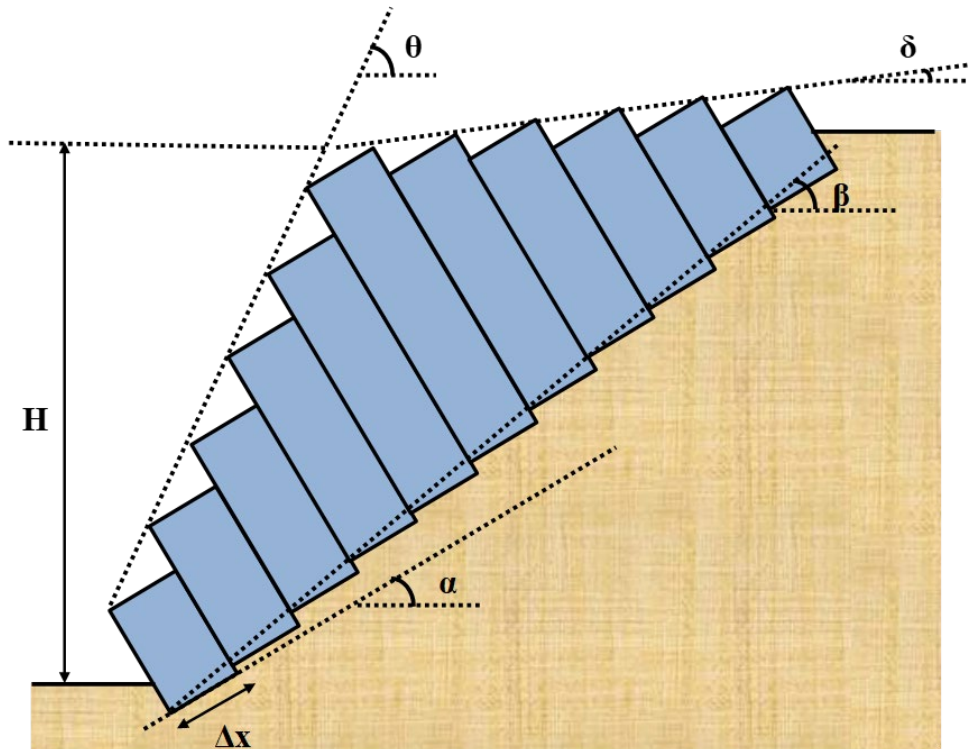
- Block n Toppling
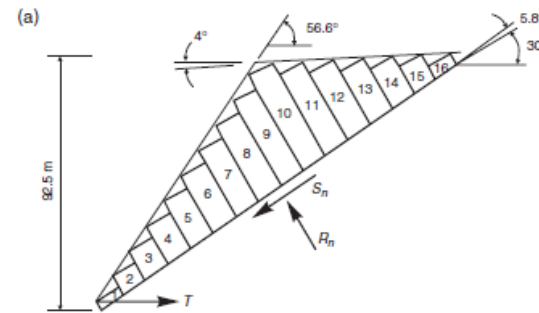
$$P_{n-1} \le \frac{P_n(M_n - \mu\Delta x) + (W_n/2)(y_n \sin\alpha - \Delta x \cos\alpha)}{L_n}$$

- Block n Sliding

$$P_{n-1} \le P_n - \frac{W_n(\mu\cos\alpha - \sin\alpha)}{1-\mu^2}$$

Block Toppling

- Block Toppling Example Calculation from Wyllie and Mah (2004)

| n	y_n	$y_n/\Delta x$	M_n	L_n	$P_{n,t}$	P_{n-s}	P_n	R_n	S_n	S_n/R_n	Mode
16	4.0	0.4			0	0	0	866	500	0.577	
15	10.0	1.0			0	0	0	2165	1250	0.577	STABLE
14	16.0	1.6			0	0	0	3463	2000	0.577	————
13	22.0	2.2	17	22	0	0	0	4533.4	2457.5	0.542	
12	28.0	2.8	23	28	292.5	−2588.7	292.5	5643.3	2966.8	0.526	T
11	34.0	3.4	29	34	825.7	−3003.2	825.7	6787.6	3520.0	0.519	O
10	40.0	4.0	35	35	1556.0	−3175.0	1556.0	7662.1	3729.3	0.487	P
9	36.0	3.6	36	31	2826.7	−3150.8	2826.7	6933.8	3404.6	0.491	P
8	32.0	3.2	32	27	3922.1	−1409.4	3922.1	6399.8	3327.3	0.520	L
7	28.0	2.8	28	23	4594.8	156.8	4594.8	5872.0	3257.8	0.555	I
6	24.0	2.4	24	19	4837.0	1300.1	4837.0	5352.9	3199.5	0.598	N
5	20.0	2.0	20	15	4637.5	2013.0	4637.5	4848.1	3159.4	0.652	G
4	16.0	1.6	16	11	3978.1	2284.1	3978.1	4369.4	3152.5	0.722	
3	12.0	1.2	12	7	2825.6	2095.4	2825.6	3707.3	2912.1	0.7855	————
2	8.0	0.8	8	3	1103.1	1413.5	1413.5	2471.4	1941.3	0.7855	SLIDING
1	4.0	0.4	4	—	−1485.1	472.2	472.2	1237.1	971.8	0.7855	
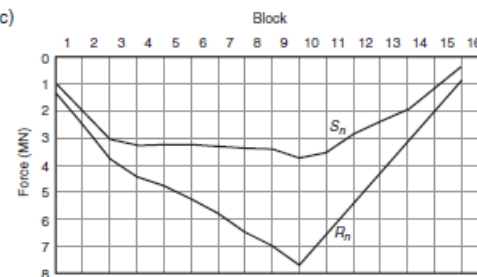
Empirical Methods

- Hoek and Bray (1981)

- Haines and Terbrugge (1991)

- Sjoberg (2001)

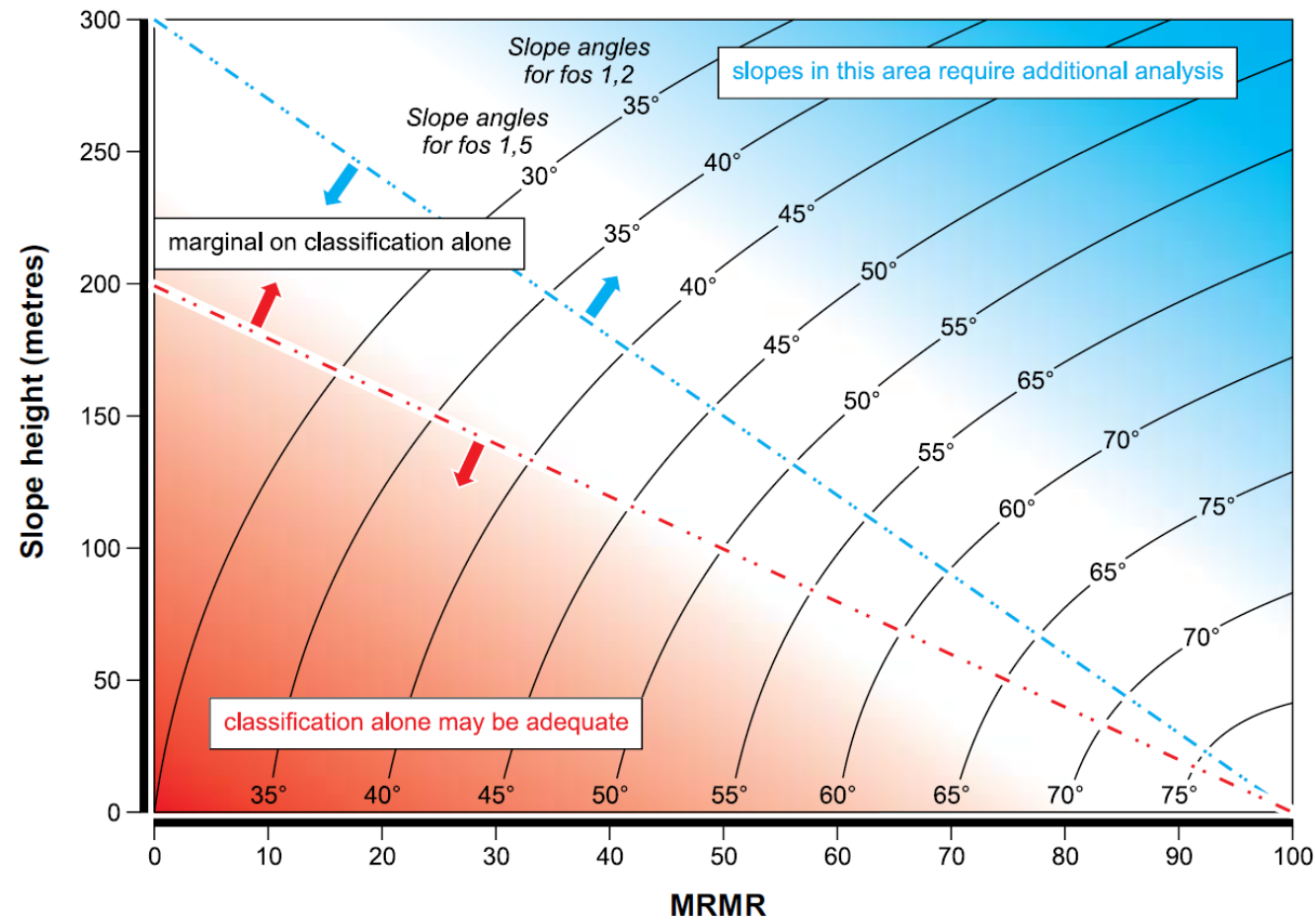- Carter and Carranza-Torres (2019)
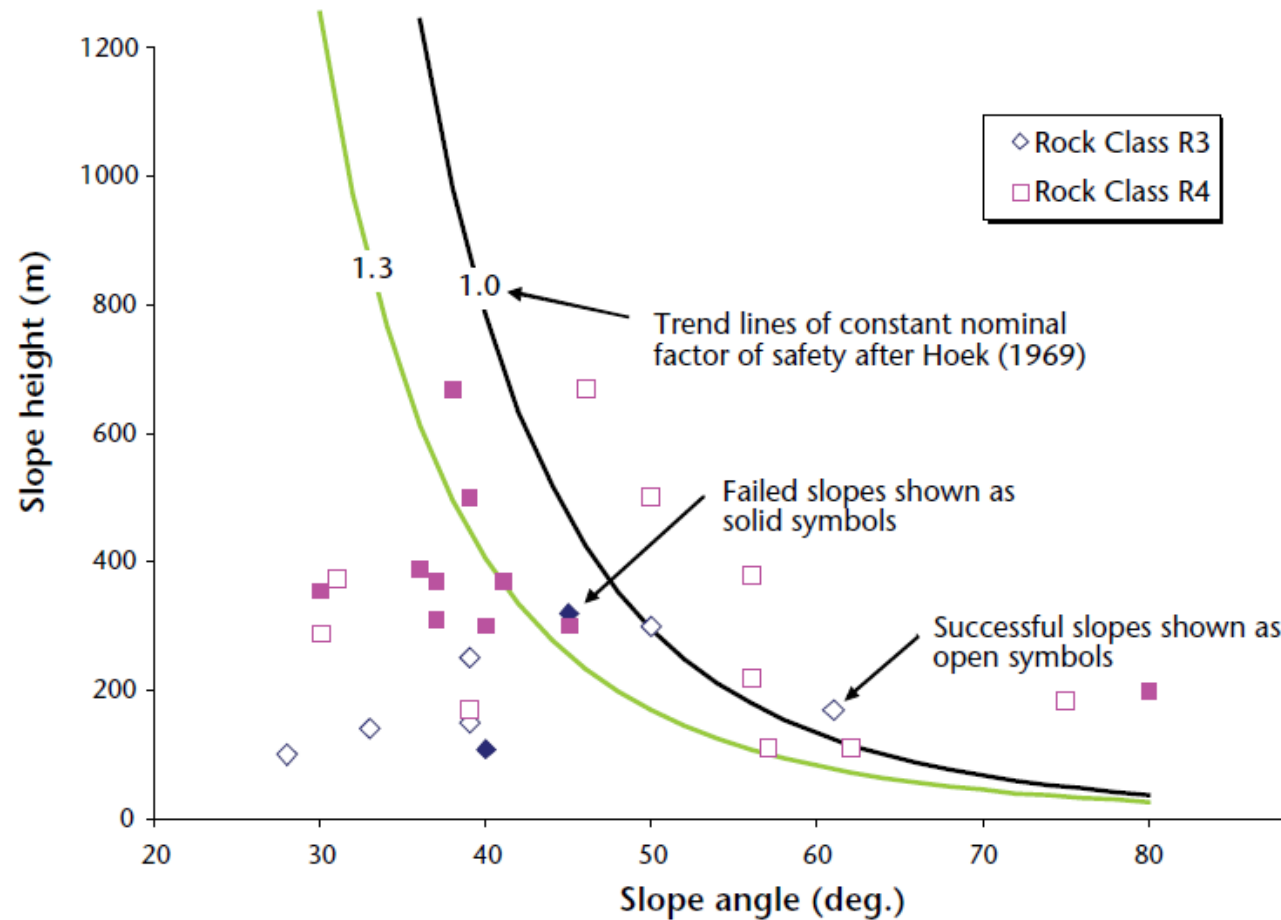
Empirical Methods

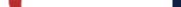- Hoek and Bray (1981).
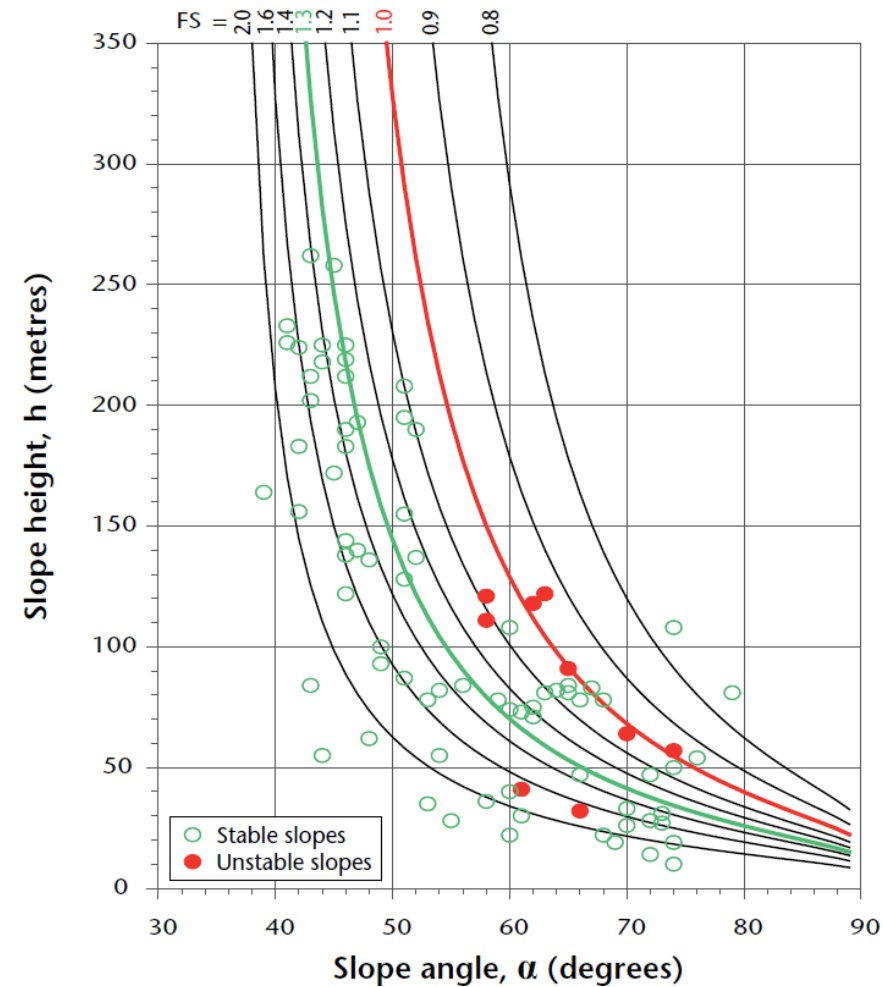
Empirical Methods

- Haines & Terbrugge (1991) Chart for Determining Slope Angle and Slope Height.

Empirical Methods

- Rock Slope Success and Failure Designated by Rock Strength from Sjoberg (2001).

Empirical Methods

- Mechanical Model Based m_b Curve Overlay to the Empirical Slope Height vs Slope Angle Plot (Carter and Carranza-Torres, 2019).

$$m_b = m_i \exp\left(\frac{GSI - 100}{28 - 14D}\right)$$
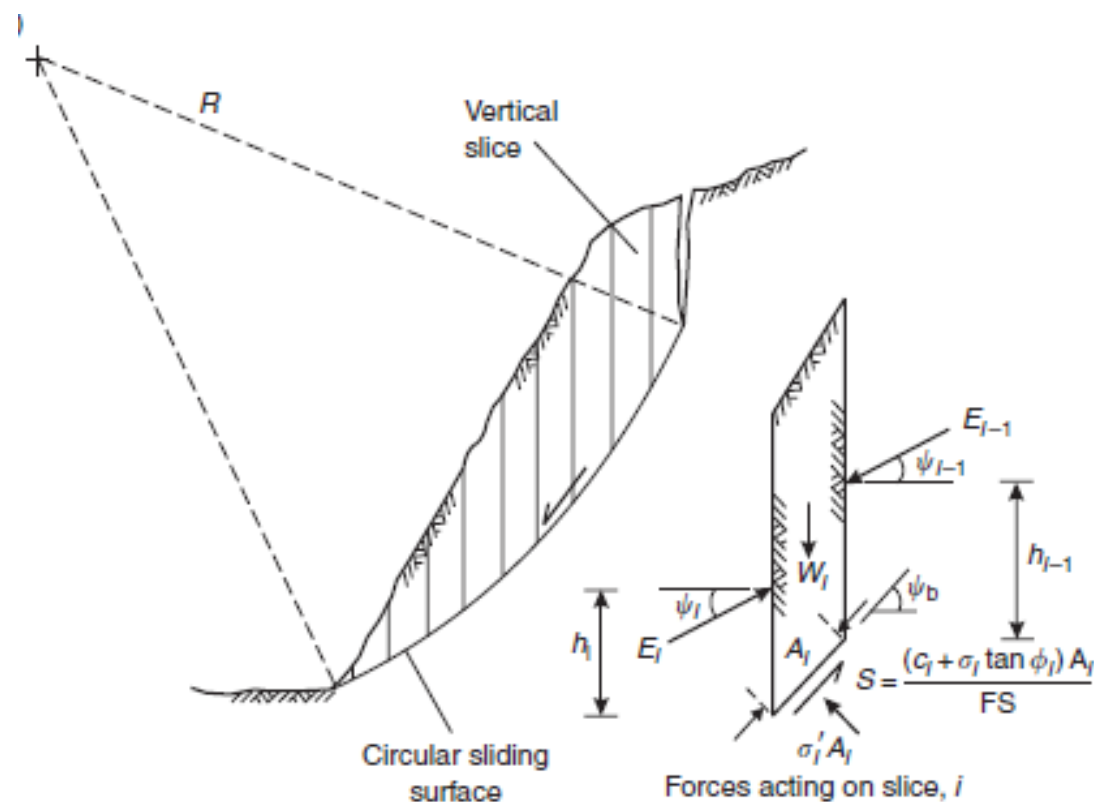
Assumed D = 0.2

Limit Equilibrium

- Basics

- Comparison of LE Methods

- Missing Physics

A practical, complete and accessible description of limit equilibrium methods is in Duncan and Wright (2005).

Duncan JM & Wright SG (2005). *Soil Strength and Slope Stability*. John Wiley & Sons, New Jersey.

Limit Equilibrium - Basics

$$ul = 0.5\gamma whl^2 \;:\; uL = 0.5\gamma wLhl + hr \;:\; ur = 0.5\gamma whr^2$$
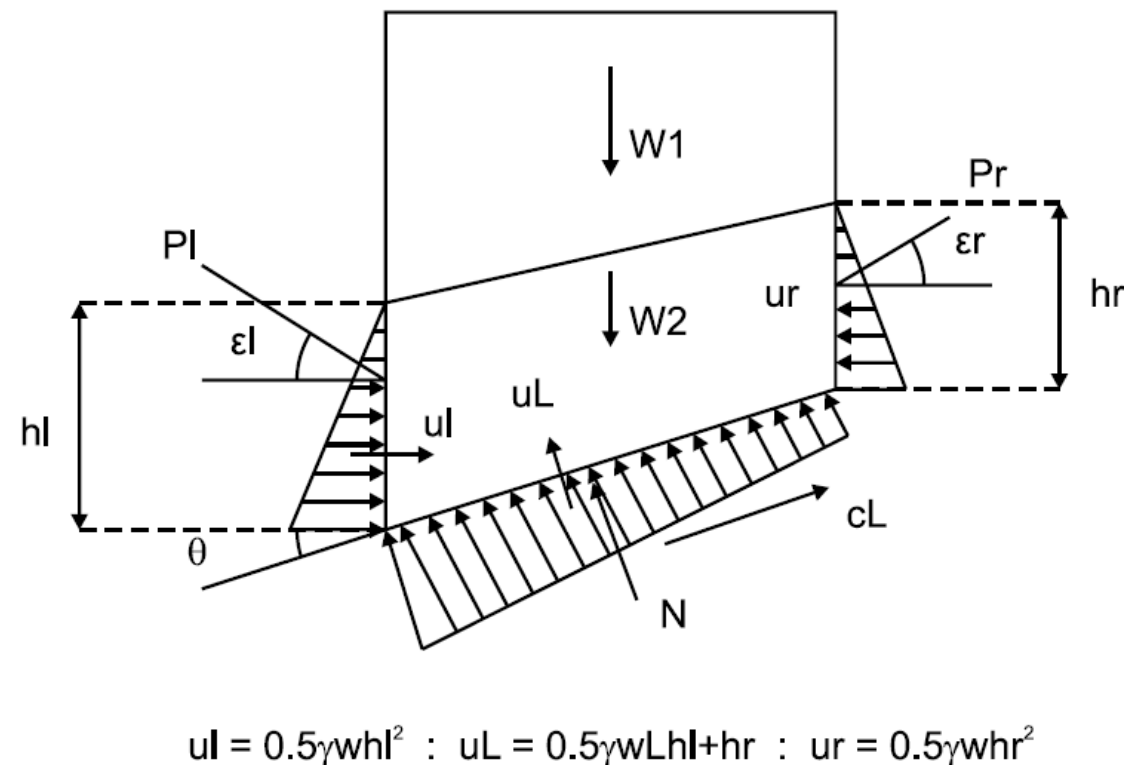
Comparison of FoS for various LE Methods

- Results for a typical homogeneous slope demonstrate that provided both force and moment equilibrium are satisfied the range in the FoS is less than 2%. This is true regardless of the search algorithm.

| Method	FOS	Difference (%)
Ordinary/Fellenius	0.629	-50.1
Bishop Simplified	1.276	1.2
Janbu simplified	1.216	-3.6
Janbu corrected	1.269	0.6
Corps of Engineers #1	1.270	0.7
Corps of Engineers #2	1.272	0.9
Spencer	1.268	0.6
Lowe-Karafiath	1.268	0.6
Sarma	1.268	0.6
Morgenstern-Price	1.261	0.0

Recommended
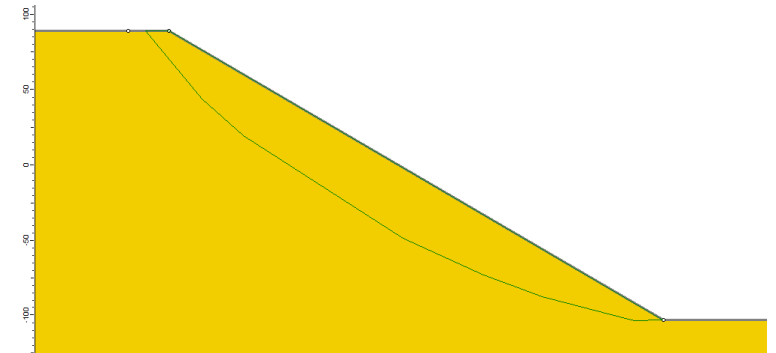
Rafiei Renani H, Martin CD (2018) Contribution to Design Acceptance Criteria for Slope Stability Analyses in Open Pits, Technical Report, Large Open Pit (LOP) Project.
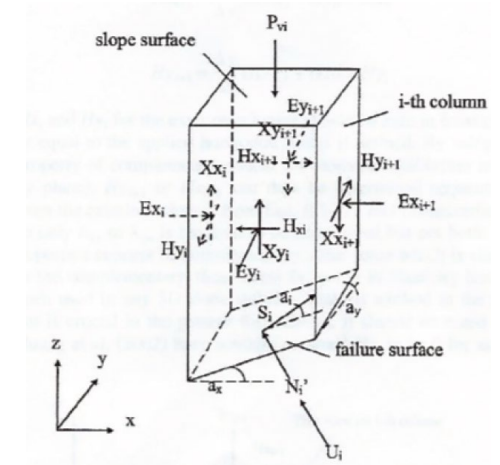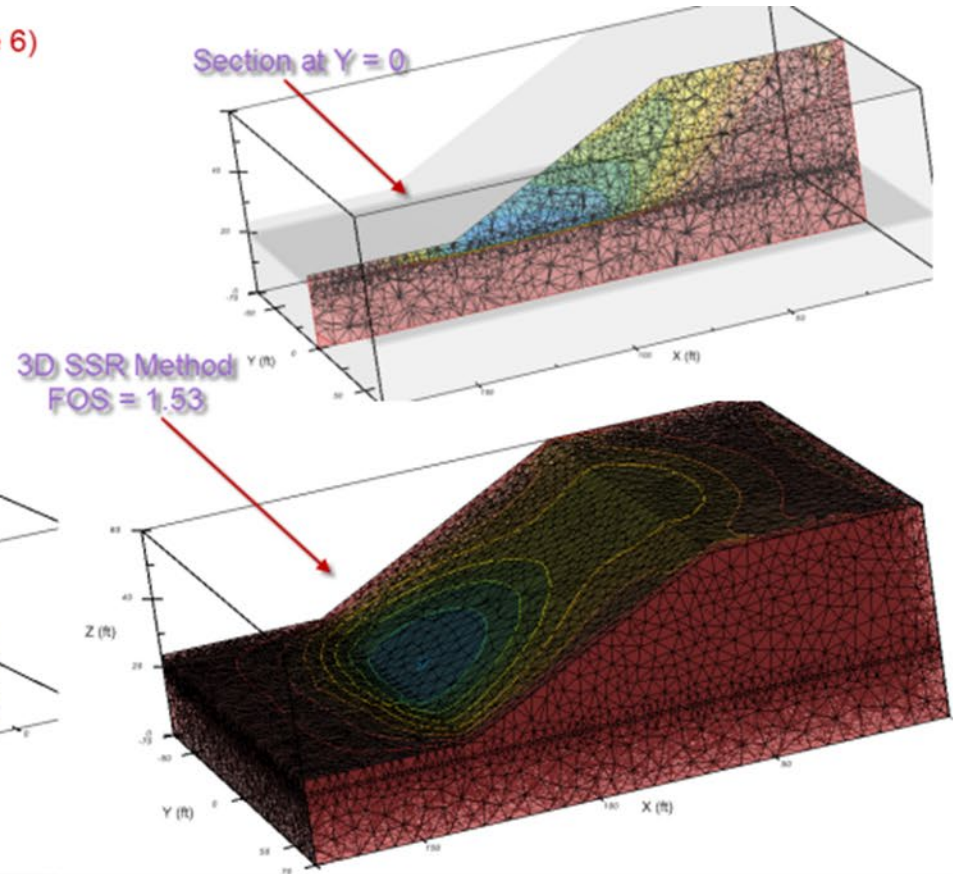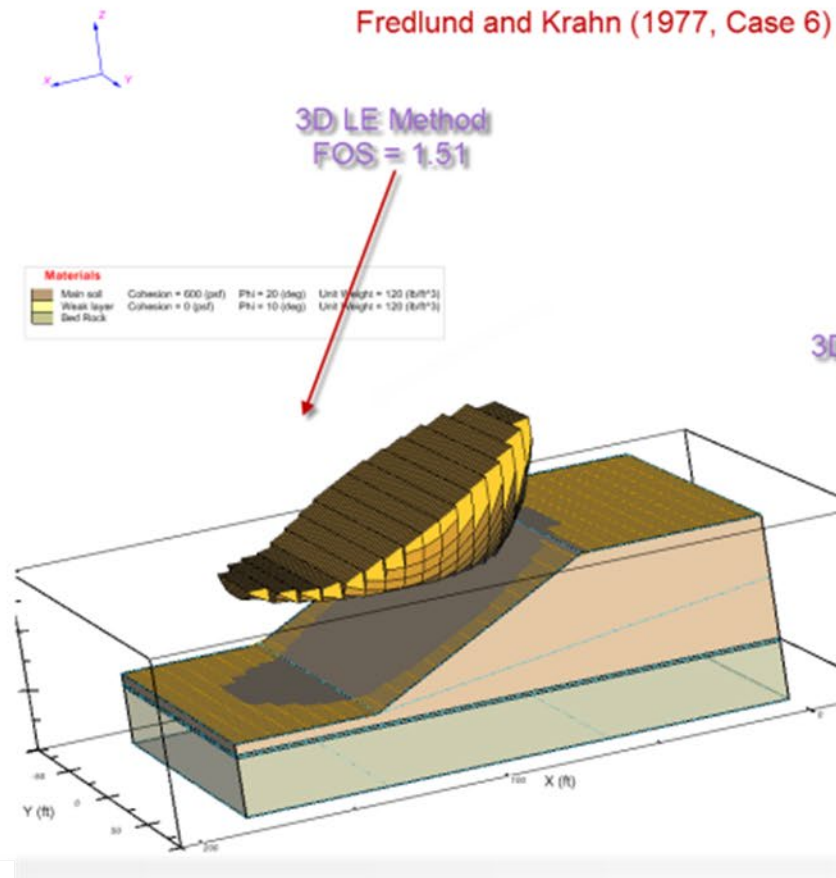
Example: Factor of Safety

- Factor of Safety computed by *Slide* using the non-vertical Sarma analysis (1.04) and Morgenstern-Price vertical slice (1.25).
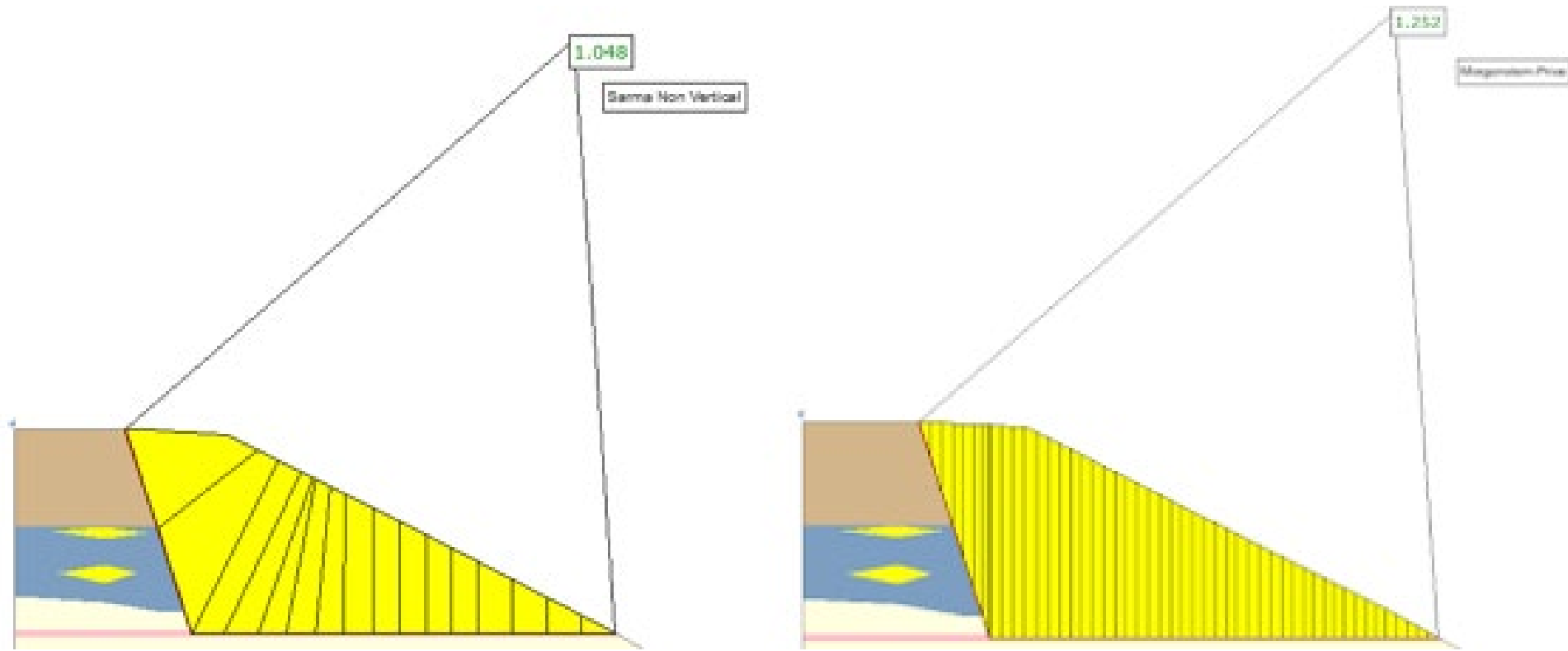
# Three-dimensional Example: Simple slope geometry

Commercial Limit Equilibrium Software

Supplier	2D	3D	Comments
RocScience	*Slide2*	*Slide3*	
Bentley	*Plaxis 2D LE*	*Plaxis 3D LE*	Formerly known as *SVSLOPE* from SoilVision Systems Ltd.
O. Hungr Geotechnical Research, Inc.		*CLARA-W*	
SEEQUENT	*Slope/W*		SEEQUENT purchased GEOSLOPE in 2019. Bentley purchased SEEQUENT?

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Limit Equilibrium: Missing physics

- Despite its popularity the LE Method of Slices has limitations.

- It is based purely on the principle of statics; that is, the summation of moments, vertical forces, and horizontal forces. The method says nothing about strains and displacements, and as a result it does not satisfy displacement compatibility. It is this key piece of missing physics that creates many of the difficulties with the limit equilibrium method.

Fig. 21. Normal stress distributions along a toe slip surface.
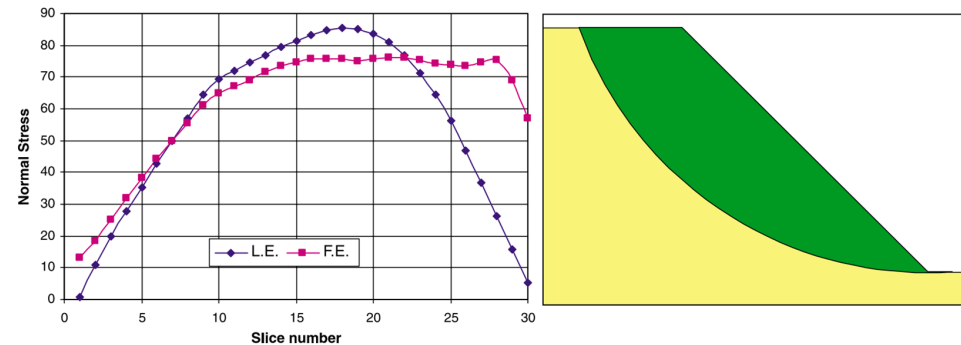

- Krahn (2003) the developer of Geoslope's program *Slope/W* proposed that *"one alternative is to set aside the whole concept of using statics and move totally to a stress–strain based approach. In other words use Finite Element Software to determine the stresses acting on the base of each slice".*

Two Major Types of Traditional Numerical Methods

- **Finite Element**

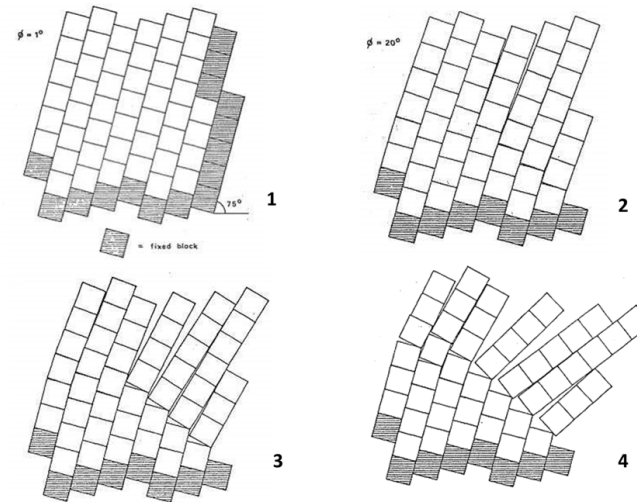 ○ (Wilson and Clough at U of California, 1960)
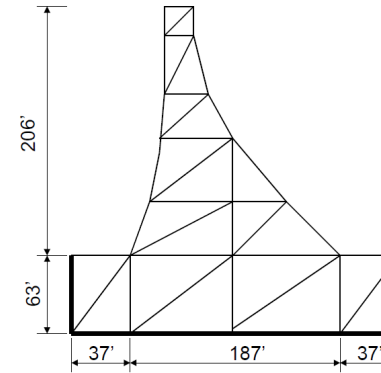
First Finite Element Mesh Used for the Analysis of Gravity Dam.

- **Finite Difference**

  ❖ Continuum

  ❖ Discontinuum

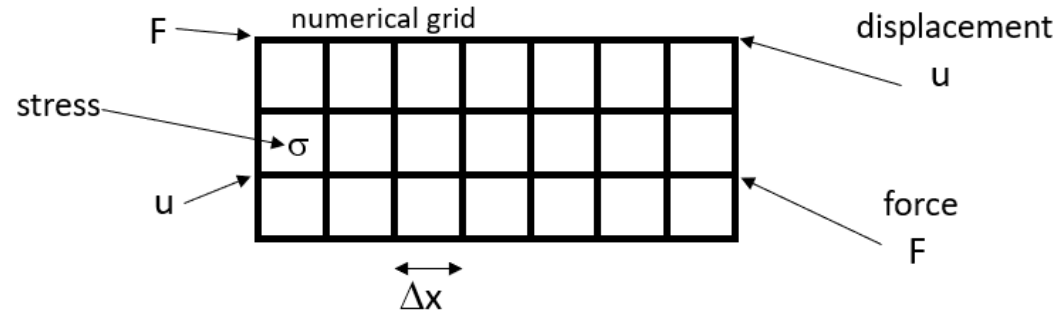 ○ (Cundall at Imperial College, 1970)

Cundall, P.A, 1971, A computer model for simulating progressive large-scale movements in blocky rock systems. *Proc. Symposium on Rock Fracture*. Nancy. France. No. PNE 5010.

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Commercial Numerical Modelling Software

Supplier	2D	3D	Comments
RocScience	*RS2*	*RS3*	Finite Element
Bentley	*Plaxis 2D*	*Plaxis 3D*	Finite Element
Itasca	*FLAC, FLAC/Slope**	*FLAC3D*	Finite Difference; Continuum
	UDEC	*3DEC*	Finite Difference; Discontinuum
Dassault Systèmes		*ABAQUS*	Finite Element

Methods of Solution in Time Domain

Finite Differences vs Finite Elements

- It may be shown, in specific cases, that the algebraic equations resulting from both formulations are identical.

- For example, the element stiffness matrices of *FLAC*´s triangles are <u>identical</u> to those for isoparametric, constant-strain finite elements.

- It´s the <u>solution scheme</u> that may be different (explicit vs implicit). Also, *FLAC* has mixed discretization for better accuracy of plastic flow.
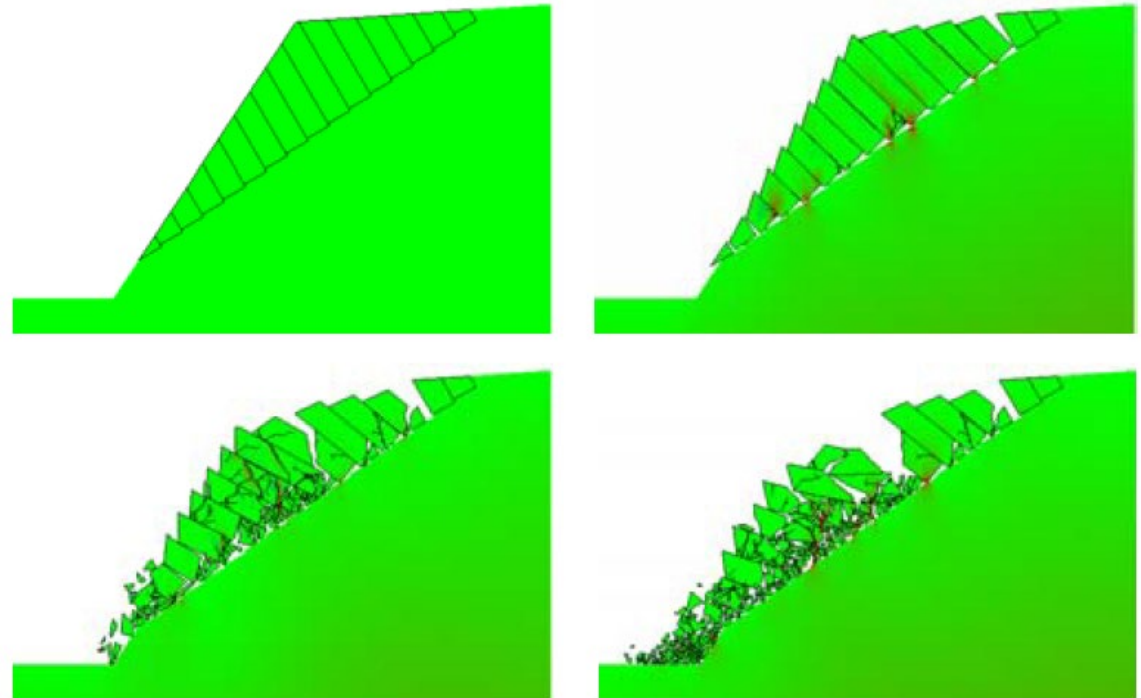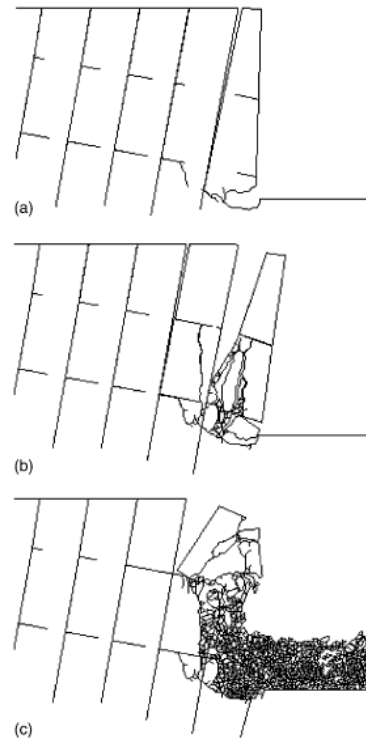
New Numerical Methods

- Hybrid Methods
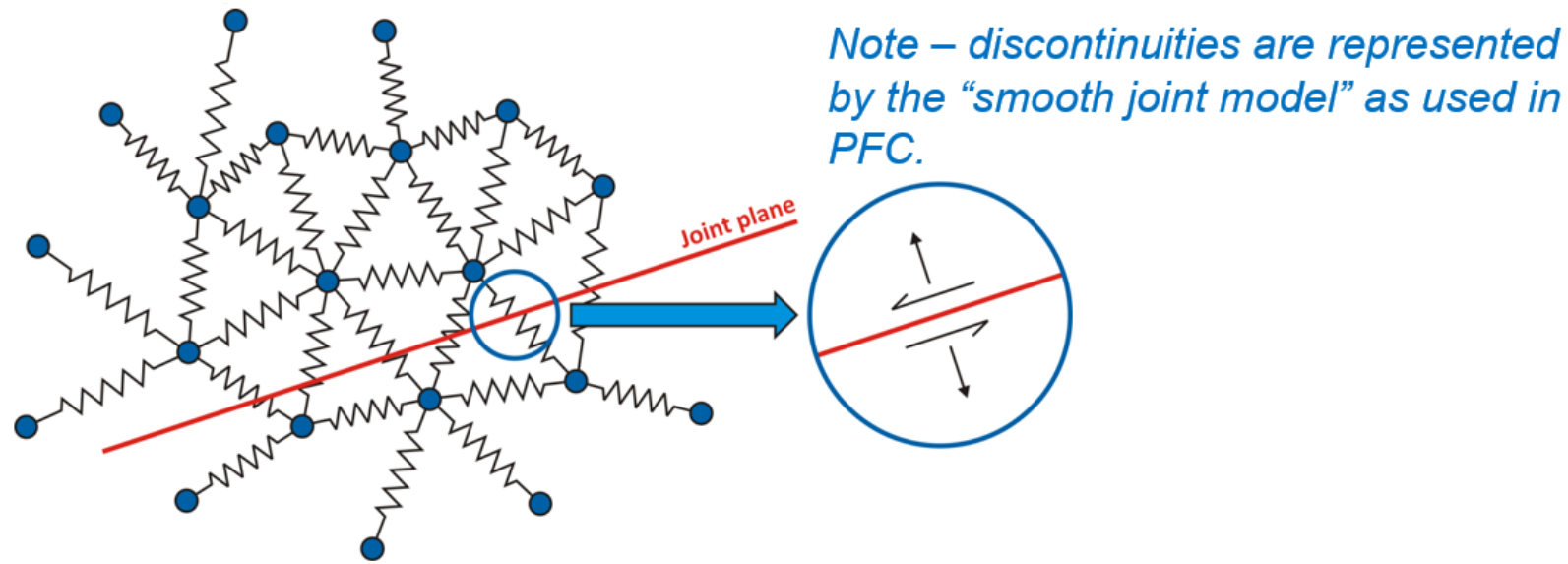
- Lattice Method

- Material Point Method

Hybrid Methods

- Transition from continuum to discontinuum following fracture.

- Often shown in 2D only.

Lattice Method

- The lattice scheme is better than both (continuum or DEM) at modeling brittle rock masses.

- It handles discontinuities and new fractures in the same way as DEM, but is 5 to 10 times faster.

- The model consists of point-masses (nodes) joined by springs.

Note – discontinuities are represented by the "smooth joint model" as used in PFC.

Joint plane

Why a Lattice Code is Different

- The lattice scheme is quite different, in several respects, from more conventional approaches (FE, FD, DEM etc).

- Other methods fill space with elements, but a lattice consists of point masses distributed in a quasi-random fashion, connected by springs.

- A continuum fracture criterion is not used (e.g., as in *ELFEN*); constitutive "laws" arise as emergent behavior (as in DEM).
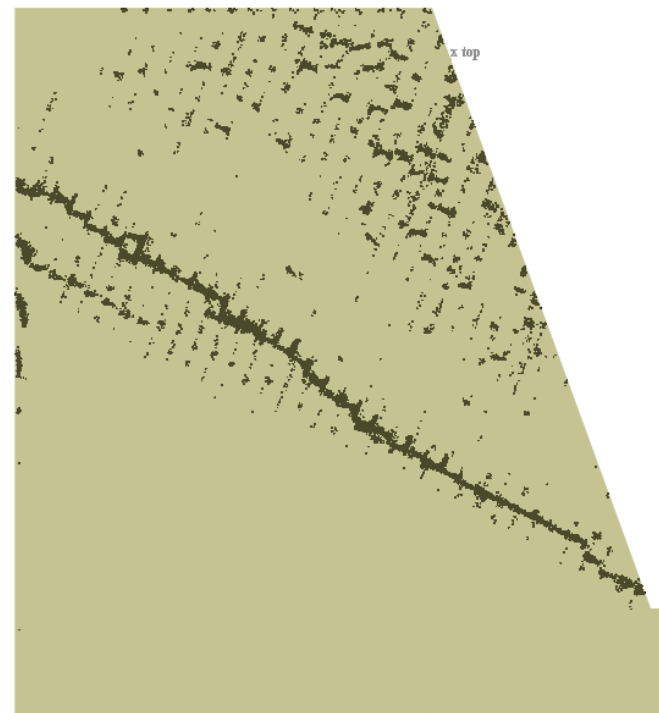
Block Toppling Example
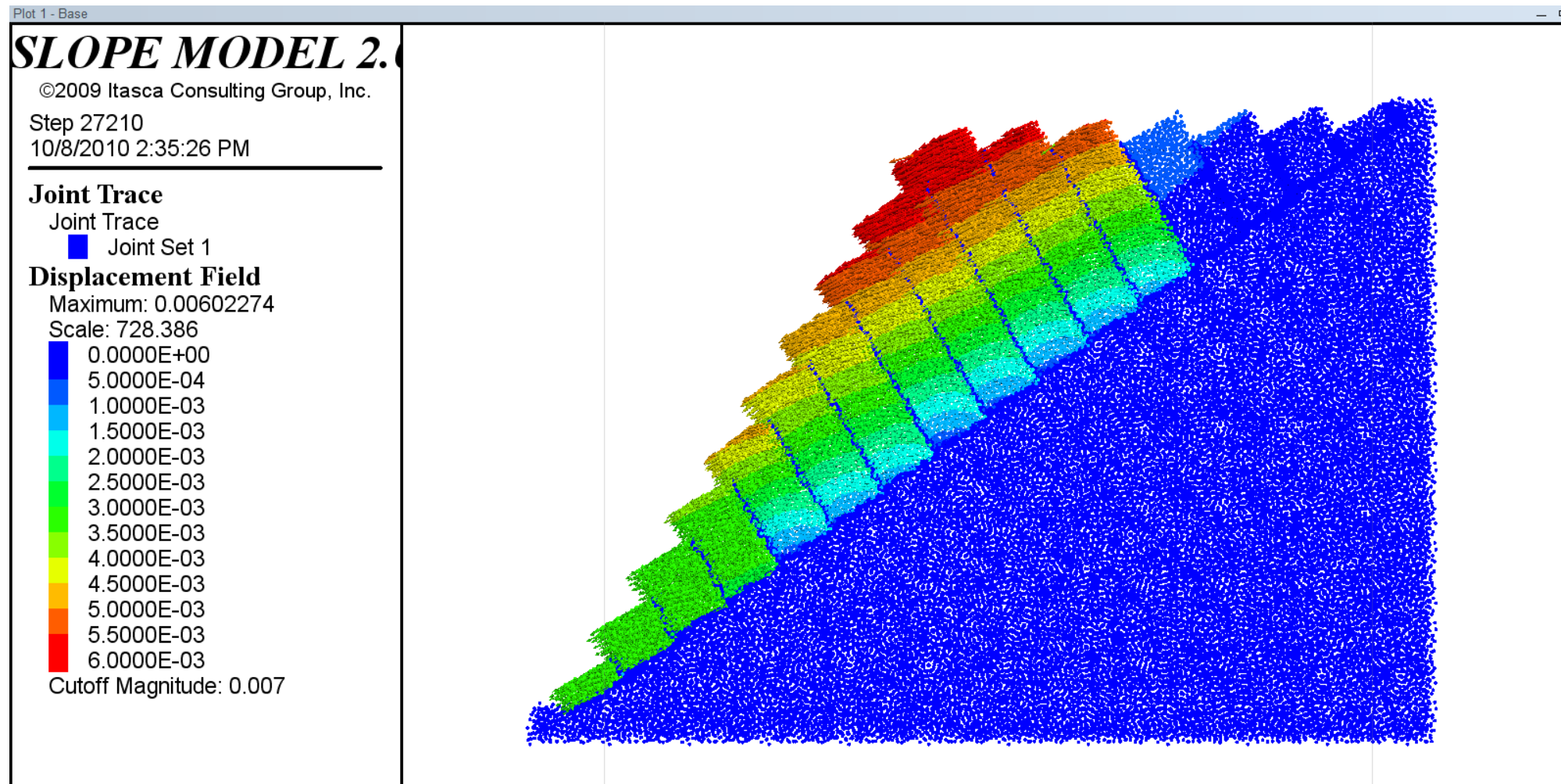
Block Toppling Example
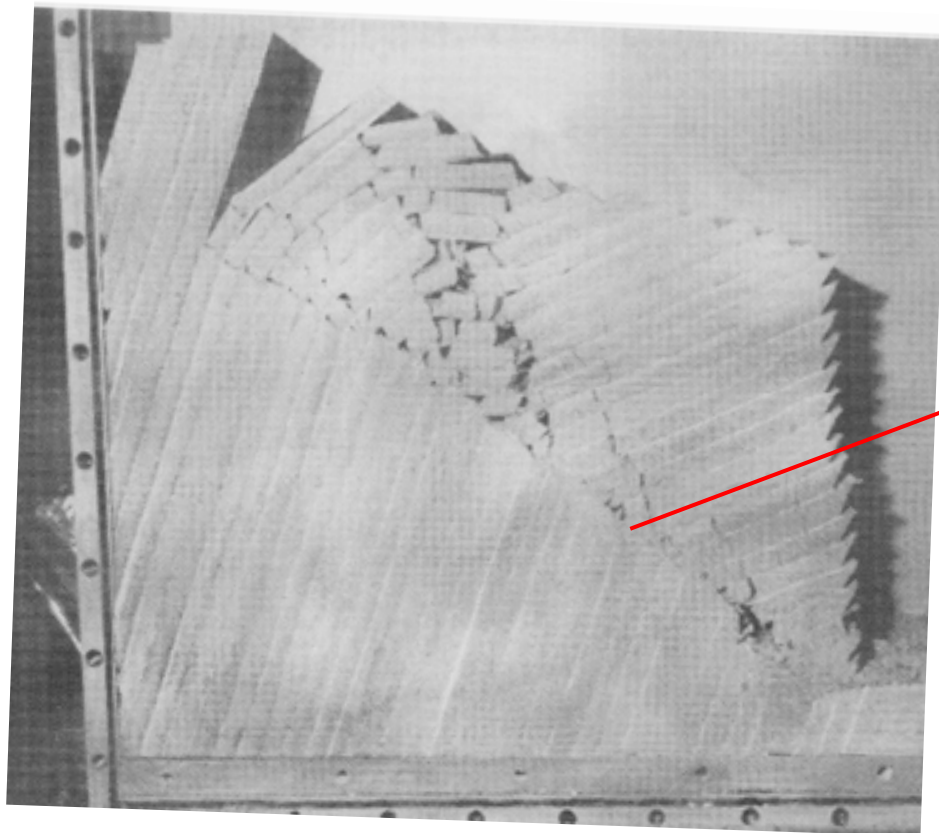
We can trace the centrifuge fracture line …
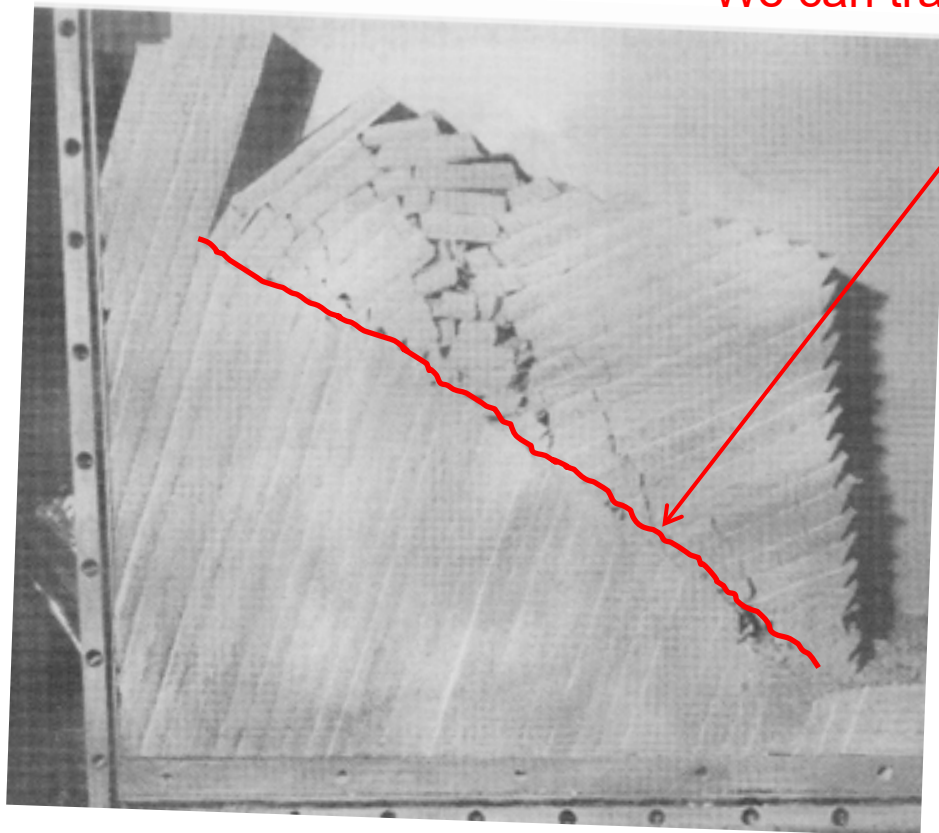
From Adhikary et al (1997), rotated to make base horizontal (Centrifuge, Test 7).

SlopeModel result.

Block Toppling Example

… and overlay it on the numerical result:

From Adhikary et al (1997), rotated to make base horizontal (Centrifuge, Test 7).

Fracture line
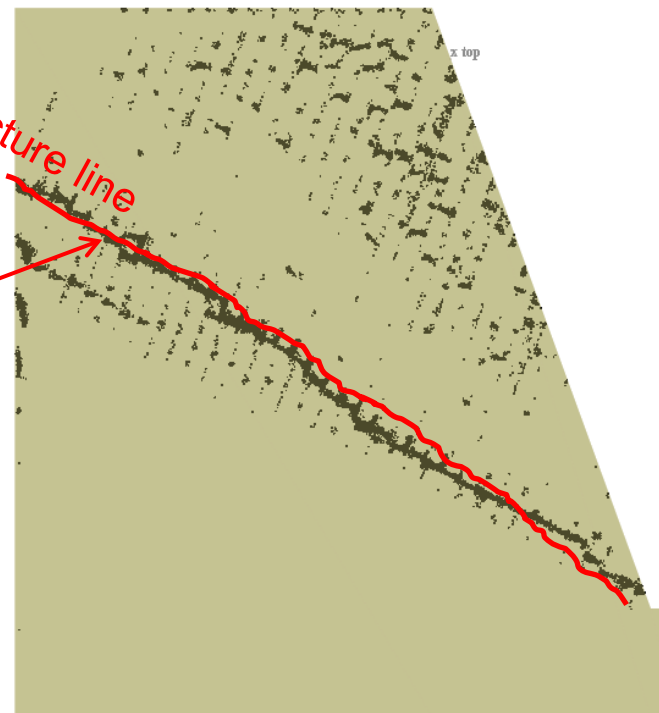
SlopeModel result.

Material Point Method

- Categorized as a meshless/meshfree or continuum-based particle method.

- Halfway between continuum and discontinuum.

- Potentially flip from "regular" zones to MPM in *FLAC3D* when large strains exist.
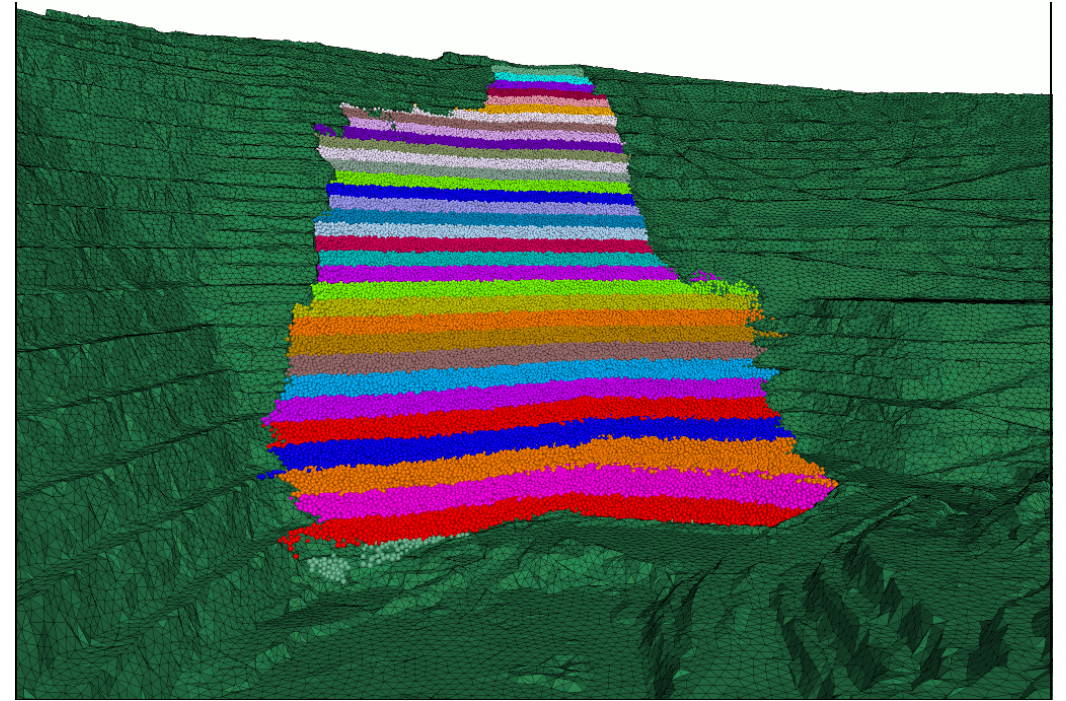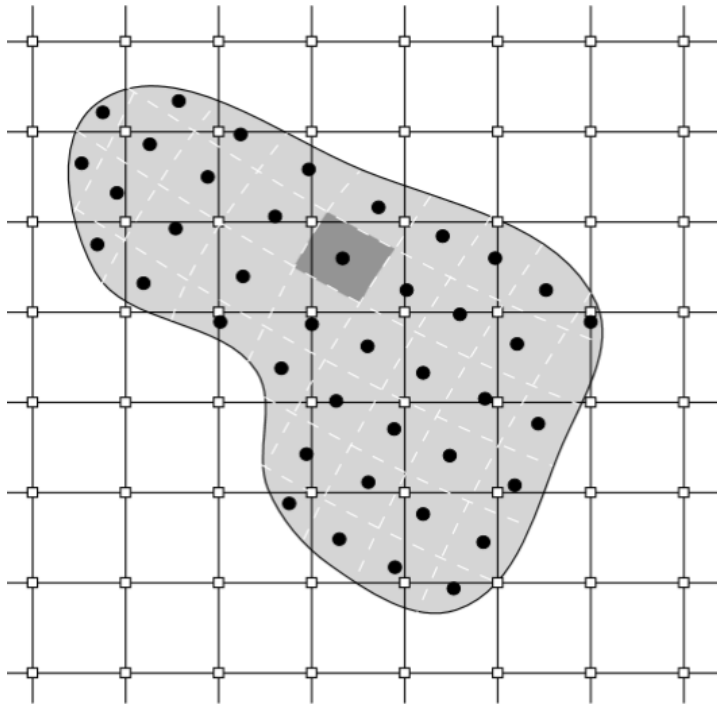
Comparison of Limit Equilibrium and Numerical Methods

- Topic of a great many journal articles and master's theses.

Factor of Safety and SRF Definition

- The **Factor of Safety** (FoS) and **Strength Reduction Factor (SRF)** are defined with respect to shear strength and is that factor by which the shear strength parameters may be reduced in order to bring the slope into a state of limiting equilibrium along a given slip surface.

$$\tau = \frac{c'}{FoS} + (\sigma - u)\frac{\tan \phi'}{FoS} \quad \text{(Limit Equilibrium)}$$

$$\tau = \frac{c'}{SRF} + (\sigma - u)\frac{\tan \phi'}{SRF} \quad \text{(Numerical Model)}$$

FoS vs SRF

LIMIT EQUILIBRIUM ANALYSES

Line of Thrust

M-P FoS=1.00
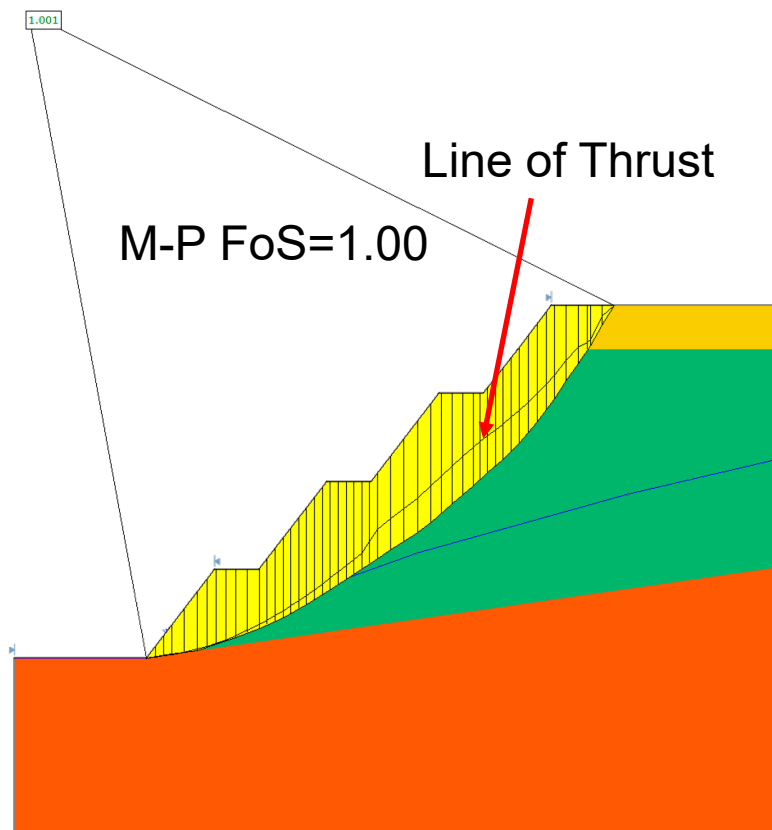
RocScience Slide 2018

FINITE ELEMENT ANALYSES RS2 SHEAR STRENGTH REDUCTION

SRF=0.97

M-P FoS=1.00 Slip Surface

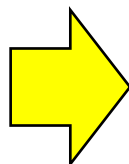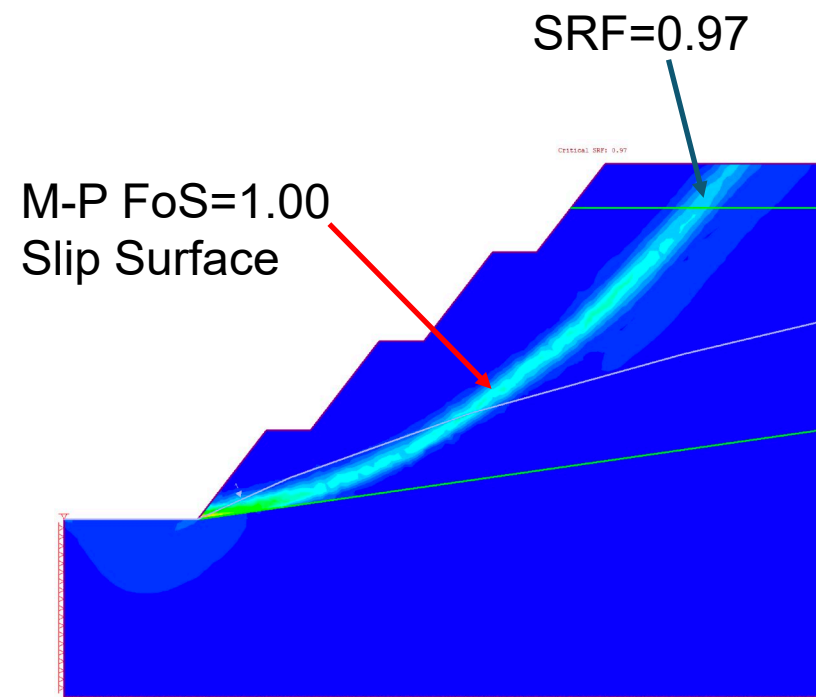RocScience RS2 2019

Comparison of FoS and SRF for simple failure mode

Method	FOS	Difference (%)
Bishop Simplified	1.276	1.2
Janbu simplified	1.216	-3.6
Janbu corrected	1.269	0.6
Corps of Engineers #1	1.270	0.7
Corps of Engineers #2	1.272	0.9
Spencer	1.268	0.6
Lowe-Karafiath	1.268	0.6
Sarma	1.268	0.6
Morgenstern-Price	1.261	0.0
Shear Strength Reduction (SRF)	1.24	-1.66
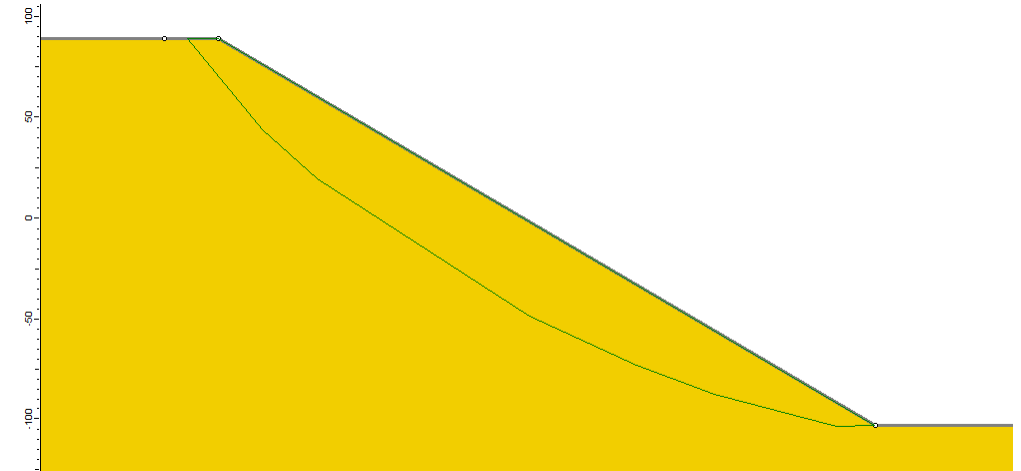
Rafiei Renani H, Martin CD (2018) Contribution to Design Acceptance Criteria for Slope Stability Analyses in Open Pits, Technical Report, Large Open Pit (LOP) Project.

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Slip Surface with a strain weakening model

Bedding plane **included** in shear strength reduction, SSR = 1.55

Bedding plane **excluded** from shear strength reduction, Critical SSR = 2.40
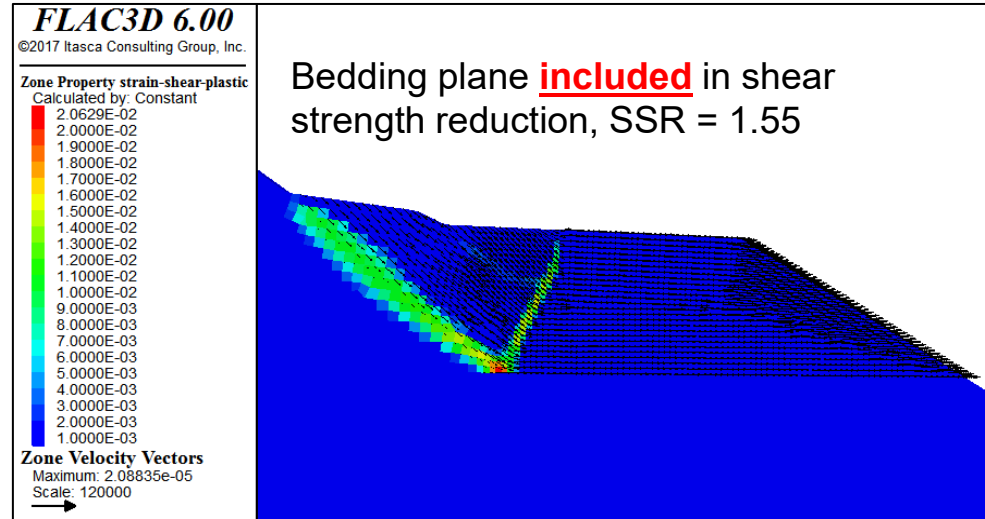
- This deep failure surface is not supported by instrumentation or observations.
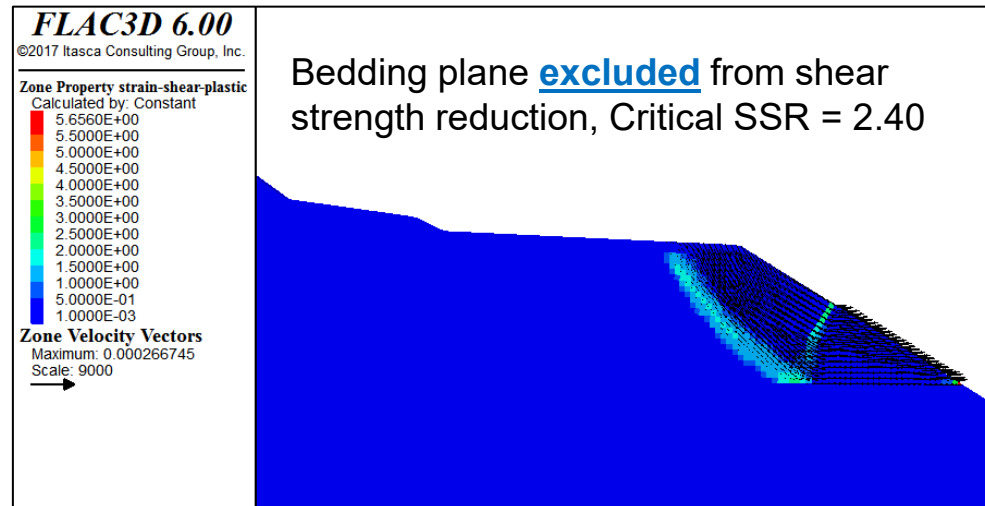
- This failure surface consistent with the location of rock mass loosening.

- **Note the Driving Wedge automatically occurs in both models. This is not possible in limit equilibrium analysis.**

Rafiei Renani H, Martin CD (2018) Stability analysis of a bedded weak rock slope, In: Proceedings of the 52nd US Rock Mechanics & Geomechanics Symposium, Seattle, US, Paper No. ARMA-2018-788.

What are Some Advantages of Using a Numerical FoS Solution?

1. Any failure mode develops naturally – no need to specify a range of trial surfaces in advance.

2. There are no restrictions on geometry – all situations (slopes, footings, tunnels, etc.) are modeled in the same way.

3. No artificial parameters (e.g., functions for inter-slice force angles) need to be given as input.

4. Multiple failure surfaces (or complex internal yielding) evolve naturally, if the conditions give rise to them.

5. Structural interaction is modeled realistically – as fully-coupled deforming elements, not simply as equivalent forces.

Important Questions

- What slope stability analysis method should be used?

- 2D or 3D?

- Continuum or discontinuum?
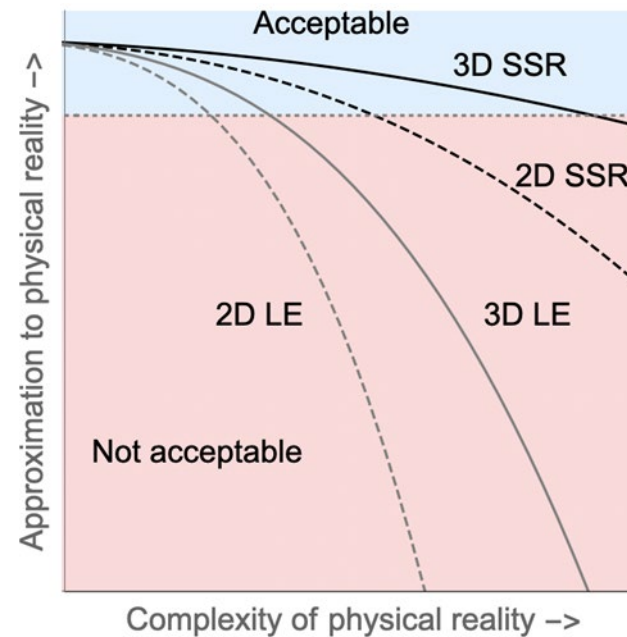
What slope stability analysis method should be used?

- Objective: Try to match method with data certainty and complexity.

Method	Stage	Data Certainty	Comment
Empirical	Conceptual and Pre-feasibility	Largely subjective	A good reality check at any stage Often used in closure studies
Analytical	All stages	Relatively high	Use stochastic or deterministic kinematic methods for bench and inter-ramp scale analysis
Limit Equilibrium	Pre-feasibility to Mine Operations	Intermediate	Most widely used method to calculate FoS of soil and rockfill embankments, including waste rock dumps and ore stockpiles
Numerical	Feasibility to Mine Operations	Significant amount of measurable data	Can simulate virtually all types of deformation and failure mechanisms

2D or 3D?

- Strictly speaking, 3D analyses are recommended/required if:

    ❖ The direction of principal geologic structures does not strike within 20° to 30° of the strike of the slope.

    ❖ The axis of material anisotropy does not strike within 20° to 30° of the slope.

    ❖ The directions of principal stresses are not parallel or not perpendicular to the slope.

    ❖ The distribution of geomechanical units varies along the strike of the slope.

    ❖ The slope geometry in plan cannot be represented by 2D analysis, which assumes axisymmetric or plane strain.

   ❖ The slope movements are not perpendicular to the strike of the slope face.

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2D or 3D?

- More detail is possible in 2D than 3D.

- 2D analysis is usually faster than 3D.

- Not always obvious what the critical 2D sections are.

- Trend is toward 3D analysis as computers become bigger/faster.

- It is often easier to perform 3D analyses than justify 2D analysis.

- 3D analyses provide best estimate of failure volumes (for runout analysis).

Continuum or Discontinuum?

- All slope stability problems involve discontinuities (faults, joints, fractures), but it is impractical to include all discontinuities in analyses.

- At a large scale slopes may appear to behave as a continuum, but what is a large scale?

- Practical solution:

  ❖ Include all faults and as many other major discontinuities as possible (often depends on time and budget).

  ❖ Treat remaining material as continuum.
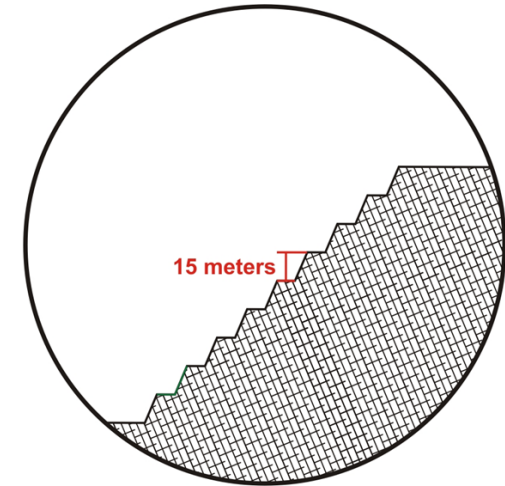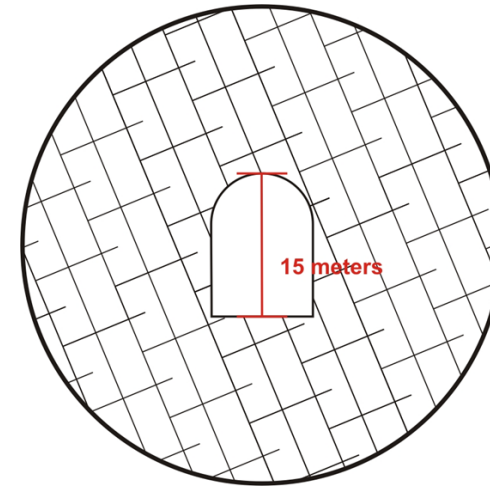
Final Thoughts (Starfield and Cundall, 1988)

- Be clear about what answers are to be answered.

- Try to identify likely modes of failure and deformation; think of experiments to perform to prove or disprove hypotheses.

- Use simplest method that will allow mechanism to occur.

- If model has limitations, perform sensitivity studies to bracket likely outcome.

- Move to more complex models only after simple models have been exhausted.
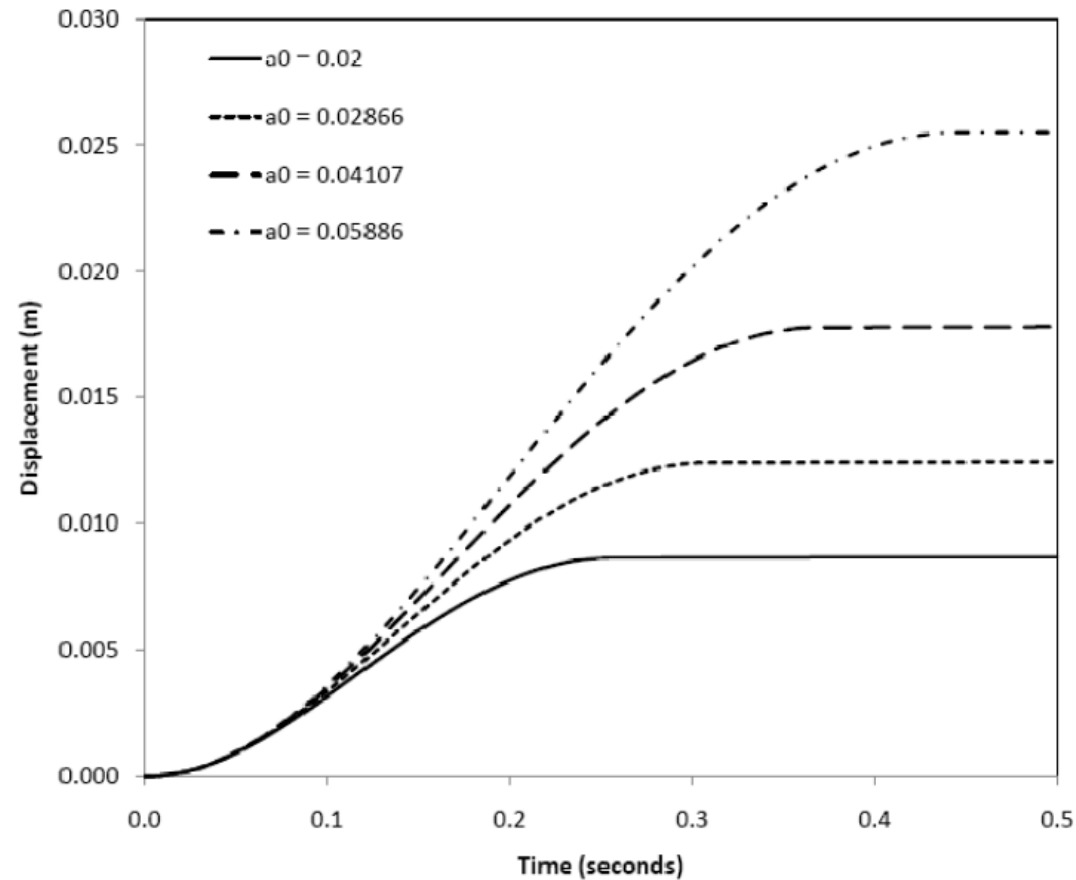
Example:

- Hydro-mechanical Response of Cracked Single-jointed Slope Subjected to Constant Infiltration (Han and Cundall, 2011).
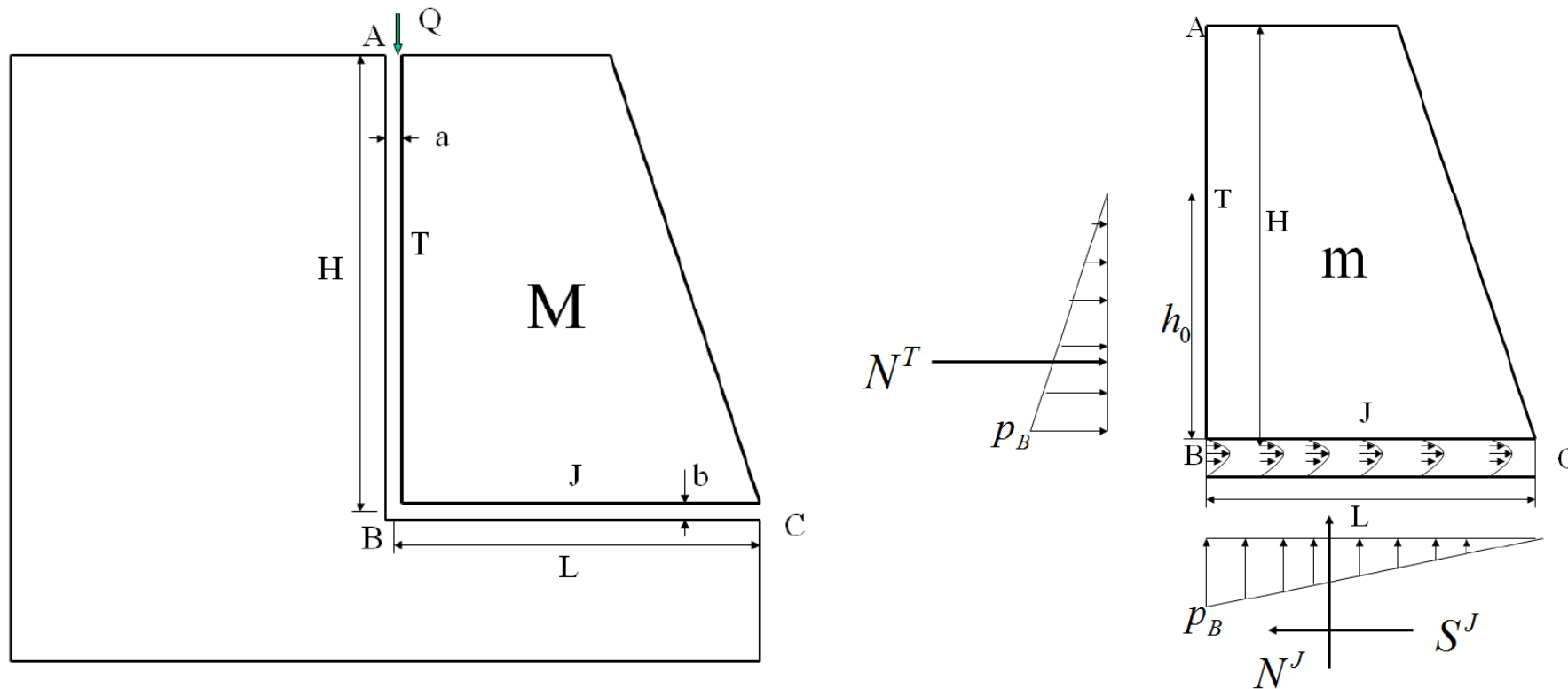
Example:

- Hydro-mechanical Response of Cracked Single-jointed Slope Subjected to Constant Infiltration.

Note: Increasing displacement with each movement episode.

Questions?

- Key references

- Physical models

- Analytical methods

- Empirical methods

- Limit equilibrium methods

- Traditional numerical methods

- New numerical methods

- Comparison of limit equilibrium and numerical methods

- Important questions

- Final thoughts

Many other topics important to slope stability analysis are **not** discussed, e.g.:
- *Rock mass characterization*
- *Strength estimation*
- *Hydrogeology*
- *Rockfall/Runout*

40 YEARS

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