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Use of a fully tensorial approach to characterize the stress variability at Forsmark

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1 INTRODUCTION

The Forsmark site is the chosen location for the repository for spent nuclear fuel in Sweden. Excavation of the repository will start in a few years, and stress magnitude and orientation are an important factor in the design. In this paper, a fully tensorial approach (Gao & Harrison 2018 a, b) is applied to stress measurement data and results obtained with an existing regional stress model, to characterize the stress variability at Forsmark site.

2 FULLY TENSORIAL APPROACH TO CHARACTERIZE THE STRESS VARIABILITY

Stress measurement data and results obtained with a regional stress model at Forsmark show that the various components of the stress tensor are significantly correlated. In such scenario, the use of a quasi-tensorial approach to characterize the stress variability is not adequate because in this approach the correlation between the stress tensor components is neglected. This correlation is taken into account in a fully tensorial approach, where a multivariate normal distribution of distinct tensor components enables to characterize the variability of stress tensors corresponding to a common Cartesian coordinate system. To determine with accuracy the correlation coefficients, a minimum number of seven stress data must be available.

The maximum likely estimation of the mean vector m_d containing the six distinct stress components s_d is calculated according to the following equation:

$$\widehat{m_d} = \frac{1}{n}\sum_{i=1}^{n} s_{d_i} = \bar{s}_d, \tag{1}$$

where n is the number of the stress data.

The probability density function f_{sd} of the multivariate normal distribution of distinct tensor components s_d is given by the following equation (Gao and Harrison 2018a):

$$f_{sd} = \frac{1}{\sqrt{(2\pi)^{\frac{1}{2}p(p+1)}|\Omega|}} exp\left(-\frac{1}{2}(s_d - m_d)^T(\Omega)^{-1}(s_d - m_d)\right), \tag{2}$$

where p is equal to the dimension of the stress tensor (equal to 2 or 3), and the estimative of the covariance matrix (Ω) is given by:

$$\Omega = cov(s_d) = \frac{1}{n}\sum_{i=1}^{n}\left(s_{d_i} - \bar{s}_d\right)\left(s_{d_i} - \bar{s}_d\right)^T. \tag{3}$$

The effective variance V_{eff} is a scalar value that measures the overall stress field dispersion and is given by the determinant of the covariance matrix Ω, according to the following equation:

$$V_{eff} = \sqrt[\frac{1}{2}p(p+1)]{|\Omega|}. \tag{4}$$

The statistical relationship between variables is formally determined by calculating their correlation coefficient ρ, which for two variables x and y is:

$$\rho = \frac{cov(x,y)}{\sqrt{var(x).var(y)}}, \tag{5}$$

where $var\ (\cdot)$ denotes the variance function. This correlation coefficient is 1 or -1 if the variables are strongly correlated and 0 if they are not correlated.

3 EXISTING STRESS MEASUREMENT DATA AND REGIONAL STRESS MODEL

The fully tensorial approach was applied to selected overcoring data from Forsmark site, presented in Martin (2007), and the results obtained with an existing regional stress model (Fig. 1) developed in *3DEC* (Itasca 2016). The model, presented in Hakala *et al.* (2019), includes the geological features at Forsmark site, including deformation zones (DZ), which are modelled as undulating best fit surfaces. The model is divided into five volumes, with the innermost encompassing all the planned underground facilities.

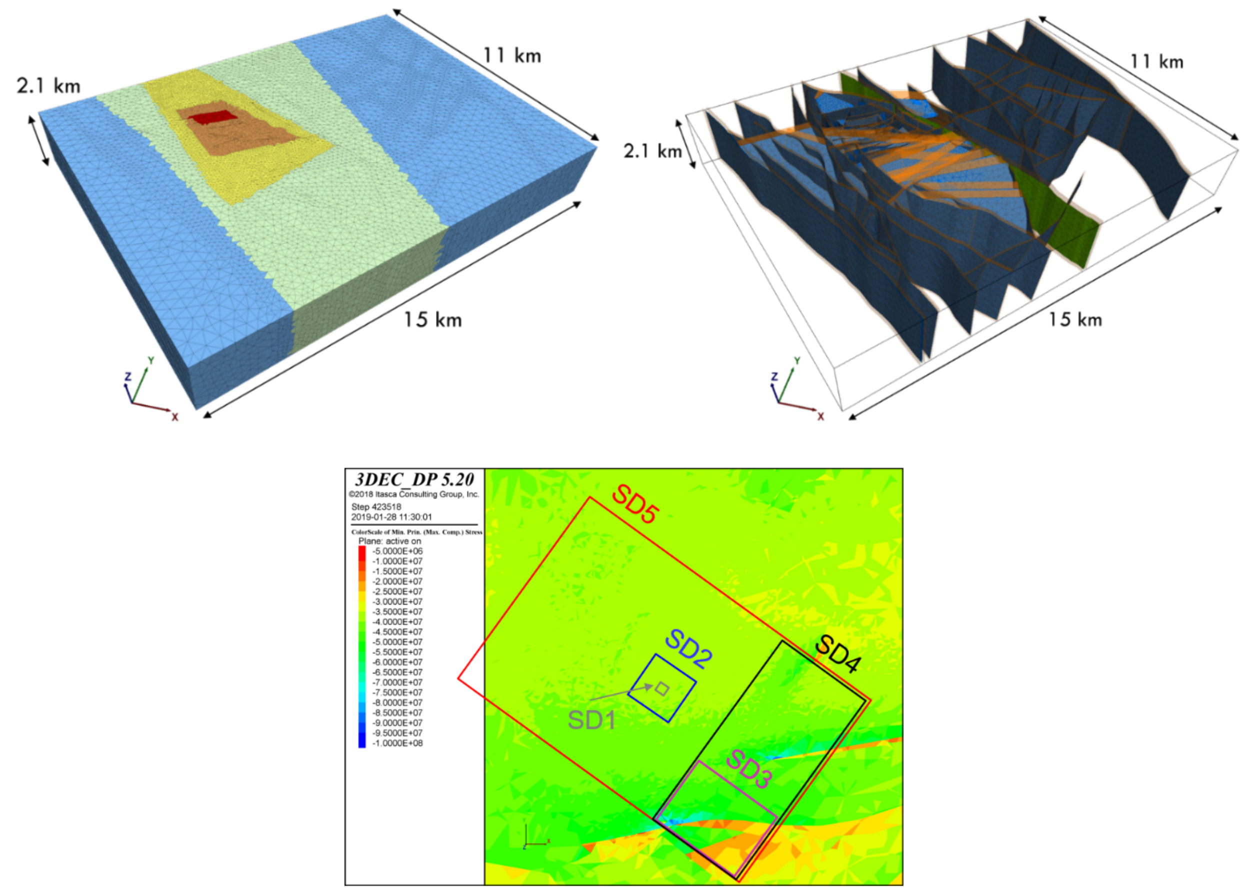

Figure 1. Regional stress model (top left) and undulating deformation zones represented by the blue, orange and green colors (top right) (extracted from Hakala *et al.* 2019) and contours of the major principal stress (σ_1) at the depth of the repository, with the sampling domains (SD) highlighted (bottom).

Two sets of calculations were conducted. In the first set, the fully tensorial approach was applied only to the stress model results, for five sampling domains, all at the target depth of the repository (470 m). Sampling domain SD1 is at the scale of the drifts with a dimension of 15 m × 15 m × 15 m. Sampling domain SD2 is at the scale of hundreds of meters with a dimension of 400 m × 400 m × 15 m. Sampling domains SD3 and SD4 are at the scale of hundreds of meters and include one and two major regions, respectively, with significant stress heterogeneity. Sampling domain SD5 is at the scale of the repository. In the second set of calculations, the fully tensorial approach was applied to both overcoring data and stress model results, which were grouped by depth ranges, including: 0–50 m, 50–100 m, 100–150 m, 150–200 m, 200–250 m, 250–300 m, 300–400 m, and 400–500 m. The stress model results were obtained exactly at the location of the stress measurements.

4 RESULTS

4.1 *Results from existing stress model and sampling domains*

4.1.1 *Euclidean mean stress field*

Figure 2 presents a histogram of the magnitude of the major principal stress and a stereographic projection of stress orientations, obtained for the sampling domains SD2 and SD3. The Euclidean mean values for the magnitude and orientation of the major principal stresses are also shown. In the histogram, the *y*-axis represents the normalized number of stress data in terms of the percentage frequency.

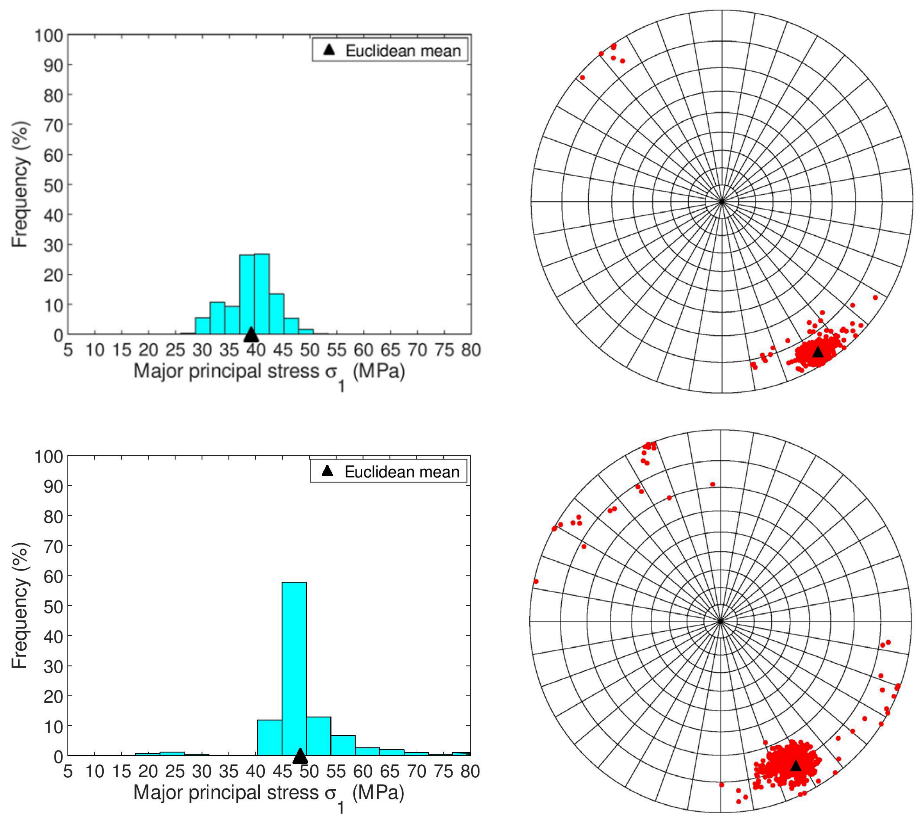

Figure 2. Magnitude (left) and orientation (right) of the major principal stress (σ_1), obtained for the sampling domains SD2 and SD3.

The effect of the physical scale on the Euclidean mean values for the magnitudes and orientations of the principal stresses was found to be fairly small. The largest differences are observed between the sampling domains with stress homogeneity (SD1 and SD2) and stress heterogeneity (SD3 and SD4). As an example, the Euclidean mean stress value of the major principal stress for SD2 and SD3 is approximately 39 and 48 MPa, respectively (Fig. 2). The mean trend of the major principal stress is not significantly affected by the stress heterogeneity and is approximately NW-SW for all the sampling domains.

4.1.2 *Stress field dispersion*

Table 1 presents the values of the effective variance for all the analyzed sampling domains. The results show that the largest values of the effective variance are observed in the sampling domains SD3 and SD4, mainly because they include zones with stress heterogeneity. In these domains, the maximum effective variance is approximately 4 MPa^2. At the scale of the sampling domain SD5, a smoothing effect in the stress field leads to values of the effective variance between those of sampling domains with stress homogeneity (SD1 and SD2) and stress heterogeneity (SD3 and SD4).

Table 1. Effective variances V_{eff}.

Sampling domain	V_{eff} [MPa^2]
SD1	0.04
SD2	0.45
SD3	4.43
SD4	2.82
SD5	1.72

4.2 *Results from stress measurement data and stress model for analyzed depth ranges*

4.2.1 *Euclidean mean stress field*

Table 2 presents the Euclidean mean values for the magnitude of the principal stresses, for all the analyzed depth ranges. The results show that, generally, the difference between the measured and calculated principal stress values is larger for the depth ranges 0–50 m, 150–250 m, and 300–400 m, compared to the other depth ranges. The orientation of the major principal stress, obtained from overcoring data and stress model results, was found to be consistent (NW-SE) for the various depth ranges.

Table 2. Euclidean mean values for the magnitude of the principal stresses obtained by using the overcoring data and stress model results: σ_1, σ_2 and σ_3, are the major, intermediate and minor principal stresses, respectively.

Depth range [m]	Overcoring data			Stress model		
	σ_1	σ_2	σ_3	σ_1	σ_2	σ_3
0-50 m	20.89	16.95	4.79	19.35	9.61	1.01
50-100 m	19.37	16.18	1.00	22.68	12.91	1.97
100-150 m	17.95	11.38	5.33	22.97	13.49	2.86
150-200 m	19.62	17.84	6.64	29.07	16.55	4.46
200-250 m	24.59	20.93	11.97	27.28	16.89	6.98
250-300 m	28.94	19.25	8.49	25.98	17.81	7.31
300-400 m	42.63	27.95	4.50	40.58	21.58	12.05
400-500 m	44.33	19.64	10.58	38.81	23.59	9.85

4.2.2 *Stress field dispersion*

The average values of the effective variance obtained for all depth ranges are found to be 11.00 and 0.04 MPa^2, for the overcoring data and the stress model results, respectively. Consequently, stress dispersion obtained for the stress model results is much less than that obtained for the overcoring data. The stress dispersion is found to be larger at shallow depths, at depth ranges 150-200 m and 200-250 m, where the effective variance ranges between approximately 15 and 16 MPa^2.

5 CONCLUSIONS

A fully tensorial approach was applied to overcoring data and the results provided by an existing 3D regional stress model, to characterize the stress variability at Forsmark site. The results show that for both the above data sets, the various stress tensor components are significantly correlated. Hence, the stress dispersion obtained with a quasi-tensorial approach would be overestimated and hence, not exclusively related with the stress heterogeneity. To apply the fully tensorial approach, a minimum number of seven stress data must be available and no significant stress gradient should be visible in the stress data. The latter condition implies that in most of the cases, the existing stress measurement data in a borehole needs to be divided in several data sets, and hence, some difficulties may arise in finding a number of seven stress data in a depth interval. This limitation can be overcome by grouping the stress data in depth ranges to compare the overall stress field dispersion at different depths. However, to characterize the stress dispersion in a rock volume, a large number of measurements may be needed, especially if the rock mass is affected by several fractures and faults. In addition, a quality control of the measurements needs to be done, to prevent that the stress dispersion is influenced by unreliable stress data. When the stress model is considered, a large number of stress data at any depth, in a rock mass volume with a thickness where the vertical stress gradient is negligible, can be considered. Thus, the stress dispersion of several sampling domains and depth intervals can be quantified. By applying the methodology to several rock regions where stress homogeneity or stress heterogeneity is assumed, it is possible to estimate the range of values for the stress dispersion. This methodology can provide information of the overall stress dispersion in each sampling domain and assist in the design of future stress measurement campaigns. However, the absolute values of the effective variances obtained with the stress model should be used with caution, because in reality the stress dispersion obtained from stress measurement data is larger. This shows that the use of both stress measurement data and stress model results is advantageous for providing some insight of the overall stress field dispersion.

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