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Fluid flow model in fractured rock by Finite Volume Black Oil Simulator (FVBOS) and *3DEC*

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1 INTRODUCTION

Investigation of multi-phase fluid flow through fractured and fissured porous media existing in rock and soil layers plays a key role in diverse applications of geotechnical engineering and energy engineering (Zienkiewicz et al. 1988, Chalaturnyk et al. 1992, Chalaturnyk & Scott 1995, Khoei et al. 2004, Zambrano-Narvaez & Chalaturnyk 2012, Azad & Chalaturnyk 2012, Salimzadeh 2018). In particular, in fractured unconventional petroleum reservoirs such as inter-crystalline sandstone, coal, carbonate and shale formations, it is vitally important to develop numerical models to analyze coupled hydro-mechanical multi-phase fluid flow in these formations considering the slippage and separation of intact rock as well as complex deformable fracture networks taking into account compressible multi-phase fluid flow governed by fluid pressure gradient, viscous forces, gravity, surface tension and capillary pressure effects as well as stress and displacement changes. Most of the numerical studies trying to simulate these behaviors have suffered from shortcomings arising from incompressible multi-phase fluid assumption, negligence of geomechanical effects, idealized fracture networks or single-phase fluid assumption without proper consideration of matrix-matrix, matrix-fracture and fracture-fracture multi-phase fluid transfer so far (Snow 1965, Barenblatt & Zheltov 1960, Kazemi et al. 1969, Damjanac 1996, Karimi-Fard et al. 2004, Sandve et al. 2012, Liu et al. 2016, Khoei et al. 2018).

A comprehensive numerical model ameliorating these deficiencies might not properly be developed in the framework of Finite Element Method (FEM) due to the complicated discontinuities encountered in both flow and displacement field due to complex deformable fracture systems existing in geological formations (Bazant & Ohtsubo 1978, Crisfield 1991, Bathe 1996, Moës et al. 1999, Moës & Belytschko 2002, Zienkiewicz & Taylor 2005, Peirce & Siebrits 2005, Nazary Moghadam et al. 2012, Nazary Moghadam et al. 2013, Hoek & Martin 2014, Hu et al. 2014, Nazary Moghadam et al. 2015). The Distinct Element Method (DEM) can be formulated by Finite Volume Method (FVM) for an assemblage of deformable blocks with no need for mesh updating (Cundall 1971, Cundall & Strack 1979, Pine & Cundall 1985, Cundall 1988, Hart et al. 1988, Zienkiewicz & Taylor 2005, Jing & Stephansson 2007). The DEM model offers a solution for the governing dynamic equations of motion taking into account complex dynamic discontinuities by defining distributed stiffness, viscous and plastic slider elements at the block contacts, employing robust contact detection and updating algorithms along with properly defined data structure. The successful application of DEM in geomechanical problems has been proven by numerous research works (Hazzard & Young 2004, Deisman et al. 2010, Hamidi & Mortazavi 2014, Khazaei et al. 2015, Shilko et al. 2015, Le et al. 2016).

The numerical formulation of coupled hydro-mechanical equations of motion for single-phase fluid flow passing through the fractures has been developed and implemented in Itasca's DEM software, *3DEC* (Damjanac 1996, Itasca Consulting Group, Inc. 2016). However, such numerical formulation has not been properly developed for the case of multi-phase fluid flow, fracture-matrix fluid transfer and large displacements so far. In this research, a numerical model has been created to address the shortcomings existing in *3DEC*, with particular application to unconventional oil and gas reservoir simulation. The Finite Volume

Black Oil Simulator (FVBOS) numerical model was developed based on discrete fracture-matrix method and was coupled with *3DEC* software. The resulting computational tool is capable of modeling coupled hydro-mechanical multi-phase fluid flow through fractures and matrix considering matrix-matrix, matrix-fracture and fracture-fracture multi-phase fluid transfer. In this model, immiscible multiphase fluid comprised of aqueous and non-aqueous liquids along with dissolved and free gas through porous matrix and fractures in rock mass is modeled by FVM using both structured and unstructured grids. The accuracy and capabilities of the developed computational tool was demonstrated by numerical examples and verification by theoretical solutions.

2 FORMULATION OF THE MODEL

To formulate the transport of multi-phase fluid in fractured porous media, the multi-phase fluid is assumed to be comprised of three immiscible phases, i.e. oil, water and gas, and the Navier-Stokes equations are simplified into Darcy's law and cubic law for fluid flow through porous matrix and fractures, respectively. Both Darcy's law and cubic law are generalized for multi-phase fluid flow considering the effects of gravitational force, pressure gradient and interfacial tension as follows (Hubbert 1956, Zimmerman & Bodvarsson 1996, Wu 2016):

$$u^{\mathscr{p}} = -\frac{K k_r^{\mathscr{p}}}{\mu^{\mathscr{p}}} \nabla (P^{\mathscr{p}} - \rho^{\mathscr{p}} g Z) \tag{1}$$

where $\mathscr{p}$ represents the fluid phase, i.e. $\mathscr{p} = \mathscr{o}$ for oil phase, $\mathscr{p} = \mathscr{w}$ for water phase, and $\mathscr{p} = \mathscr{g}$ for gas phase. The parameters $P^{\mathscr{p}}$, $\mu^{\mathscr{p}}$ and $\rho^{\mathscr{p}}$ respectively denote the pressure, viscosity and specific mass of the fluid phase $\mathscr{p}$, while g and Z are respectively gravitational acceleration and vertical coordinate aligned with the direction of gravitational acceleration. In addition, the parameter *K* denotes absolute permeability, while $k_r^{\mathscr{p}}$ is the relative permeability of the fluid phase $\mathscr{p}$. For fluid flow through fractures, the absolute permeability is defined as follows:

$$K = \frac{e^2}{12} \tag{2}$$

where *e* is the hydraulic aperture. Assuming that gas is the only fluid phase dissolved in other fluid phases, mass conservation equations are derived in differential form as follows:

$$\frac{\partial}{\partial t}\left(\frac{S^{\mathscr{w}} \phi}{B^{\mathscr{w}}}\right) + \nabla.\left(\frac{u^{\mathscr{w}}}{B^{\mathscr{w}}}\right) + \frac{q^{\mathscr{w}}}{\rho_{SC}^{\mathscr{w}}} = 0 \tag{3}$$

$$\frac{\partial}{\partial t}\left(\frac{S^{\mathscr{o}} \phi}{B^{\mathscr{o}}}\right) + \nabla.\left(\frac{u^{\mathscr{o}}}{B^{\mathscr{o}}}\right) + \frac{q^{\mathscr{o}}}{\rho_{SC}^{\mathscr{o}}} = 0 \tag{4}$$

$$\frac{\partial}{\partial t}\left[\left(\frac{S^{\mathscr{g}}}{B^{\mathscr{g}}} + \frac{R_S^{\mathscr{w}} S^{\mathscr{w}}}{B^{\mathscr{w}}} + \frac{R_S^{\mathscr{o}} S^{\mathscr{o}}}{B^{\mathscr{o}}}\right)\phi\right] + \nabla.\left(\frac{u^{\mathscr{g}}}{B^{\mathscr{g}}} + \frac{R_S^{\mathscr{w}} u^{\mathscr{w}}}{B^{\mathscr{w}}} + \frac{R_S^{\mathscr{o}} u^{\mathscr{o}}}{B^{\mathscr{o}}}\right) + \frac{q^{\mathscr{g}}}{\rho_{SC}^{\mathscr{g}}} = 0 \tag{5}$$

where ϕ is effective porosity, $S^{\mathscr{p}}$ is saturation of the fluid phases, $\rho_{SC}^{\mathscr{p}}$ is the density of the fluid phase $\mathscr{p}$ in standard temperature and pressure condition, $q^{\mathscr{p}}$ denotes the mass source or sink of fluid phase $\mathscr{p}$, $B^{\mathscr{p}}$ is reservoir volume factor of fluid phase $\mathscr{p}$ which is the ratio of the volume of fluid phase $\mathscr{p}$ in reservoir condition (RC) to that in standard condition (SC). Also, the parameters $R_S^{\mathscr{w}} = V_{SC}^{D\mathscr{g}\ in\ \mathscr{w}} / V_{SC}^{\mathscr{w}}$ and $R_S^{\mathscr{o}} = V_{SC}^{D\mathscr{g}\ in\ \mathscr{o}} / V_{SC}^{\mathscr{o}}$ are the solubility of gas in water and oil where $V_{SC}^{D\mathscr{g}\ in\ \mathscr{w}}$ and $V_{SC}^{D\mathscr{g}\ in\ \mathscr{o}}$ represent the volume of dissolved gas in water and oil in standard condition, respectively. The saturations of the fluid phases satisfy the following equation:

$$S^{\mathscr{w}} + S^{\mathscr{o}} + S^{\mathscr{g}} = 1 \tag{6}$$

There exist pressure differences between fluid phases represented by the capillary pressure concept in terms of oil-water capillary pressure $P_c^{\mathscr{o}\mathscr{w}}$ and gas-oil capillary pressure $P_c^{\mathscr{g}\mathscr{o}}$ as follows:

$$P^{\mathscr{o}} - P^{\mathscr{w}} = P_c^{\mathscr{o}\mathscr{w}} \tag{7}$$

$$P^{\mathscr{g}} - P^{\mathscr{o}} = P_c^{\mathscr{g}\mathscr{o}} \tag{8}$$

The derived governing differential equations for multi-phase fluid flow are integrated over finite control volumes, representing fractures and porous matrix, using the single point Gaussian numerical integration method. The spatially discretized equations are then discretized in time domain using an implicit pressure-explicit saturation (IMPES) method. The numerical formulation was implemented into the object oriented C++ Finite Volume Black Oil Simulator (FVBOS) code. FVBOS provides a compatible computational tool to be hydro-mechanically coupled with distinct element solid mechanics simulators. With the aid of this compatibility, the FVBOS code was coupled with *3DEC* software using 2-way coupling scheme to simulate coupled hydro-mechanical multi-phase fluid flow through fractured porous media. In this coupling method, overall pressure field which is the weighted average of pressures of the three phases calculated by FVBOS is transferred to the solid mechanics model in *3DEC* and the apertures calculated by *3DEC* is transferred to FVBOS. This iterative pressure-aperture transfer is performed until a predefined tolerance limit is reached.

3 RESULTS AND DISCUSSION

To investigate the capabilities of the developed computational tool, different numerical simulations were carried out and the results were compared with theoretical solutions. In the first series of numerical simulations, the incompressible oil-water two phase flow was modeled for two separate cases of porous matrix and single fracture and the numerical results were compared with the Buckley-Leverett theoretical solution (Buckley & Leverett 1942) as it is exhibited in Figure 1. In these numerical models, the Buckley-Leverett assumptions, which are incompressible fluid phases, incompressible porous media, constant fluid phases viscosity, unidirectional flow, and negligible gravitational forces and capillary pressure, were applied. To this end, a large value was assigned to bulk modulus of porous matrix and normal stiffness of fracture so that negligible changes in volume of porous matrix and aperture of the fracture can occur due to pressure change during the analyses. The assumption of incompressible fluid phases was also satisfied by assigning unit reservoir volume factor to the fluid phases. The values assigned to other parameters are summarized in Figure 1. As it can be observed, the numerical results generated by the developed numerical tool is in a good agreement with the Buckley-Leverett theoretical solution demonstrating the validity and accuracy of the developed model.

In the next series of numerical simulations, the air-water two phase flow through porous matrix containing a single fracture was modeled as it is illustrated in Figure 2. In this model, it was assumed that the fracture is restrained at the top and bottom so that it cannot propagate at the tips (Perkins & Kern 1961). In this model, the upper half of the fracture was modeled due to the symmetry about XY plane. The pore spaces in porous matrix and fracture was assumed to be 90 percent saturated by air and the rest of the pore space is assumed to be occupied by the irreducible water with 10 percent saturation. Water was injected at the rate of 1.0 L/s into the fractured porous media from one side of fracture considering the leakoff of the injected fluid into the porous matrix. In these analyses, the gravitational forces and capillary pressure are not taken into account and the shear stiffness, cohesion, frictional resistance and tensile strength of the fracture are neglected to be consistent with the assumptions made by Perkins & Kern (1961) for the fracture. Material parameters of porous matrix, fracture and fluid phases are given in Table 1.

Table 1. Material parameters of intact rock, fracture and fluid.

Porous Matrix	
Young Modulus (GPa)	1
Poison's Ratio	0.2
Absolute permeability (Darcy)	0.01
Porosity	0.2
Fracture	
Normal stiffness (GN/m)	1
Aperture at Zero Normal Stress (mm)	0.05
Maximum Aperture (mm)	10
Residual Aperture (mm)	0.05
Fluid	
Dynamic viscosity of water (Pa.s)	0.001
Dynamic viscosity of air (Pa.s)	0.0002

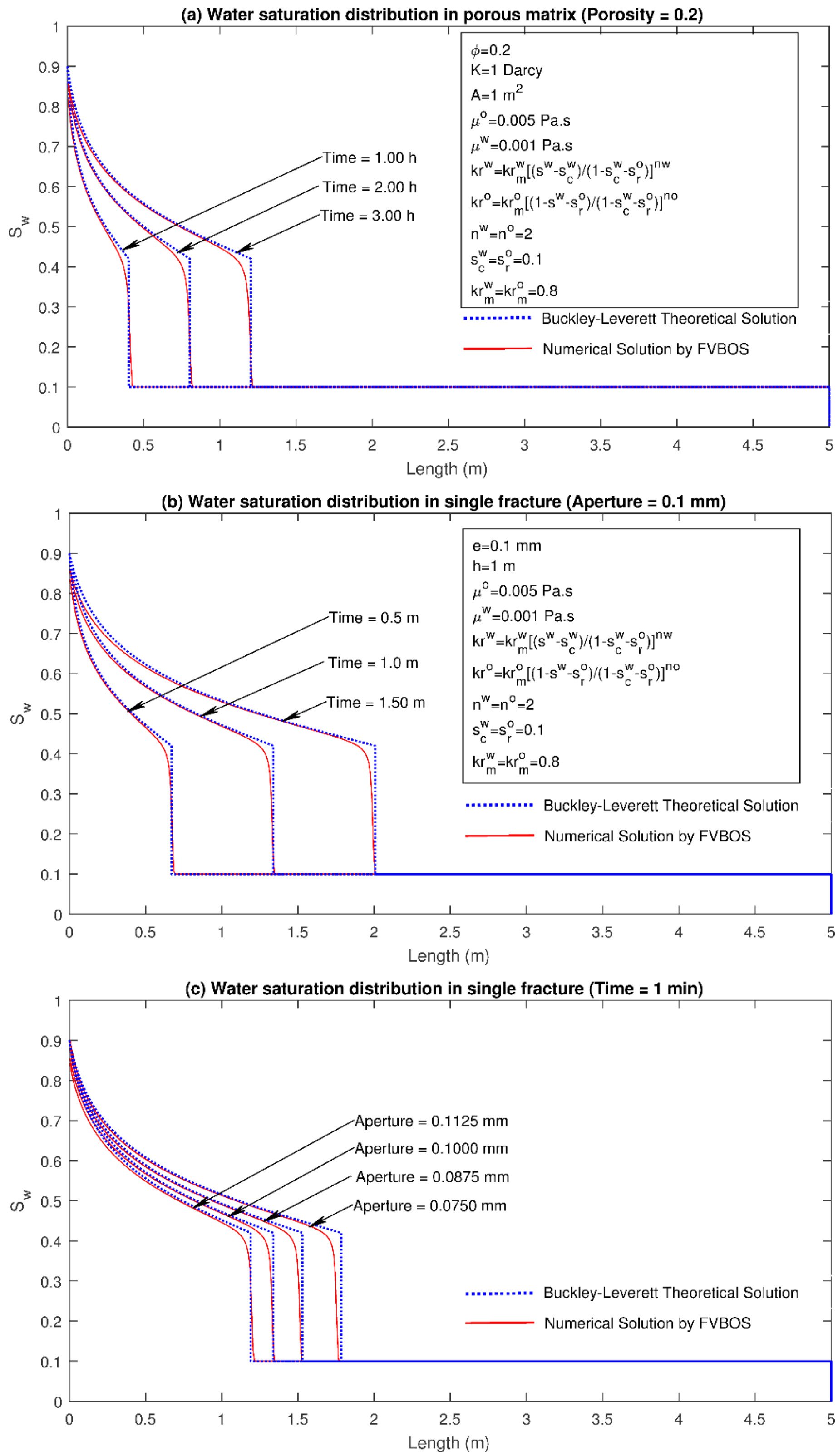

Figure 1. Verification of the developed numerical model with Buckley-Leverett theoretical solution of incompressible oil-water two phase flow for two cases of porous matrix and single fracture.

Analyses results generated by the developed numerical tool are displayed in Figure 2 in terms of pressure, fracture aperture and water and air saturation distribution as well as velocity of water and air. As it can be seen, the fracture aperture increases from zero at the fracture tip towards the fracture center where the maximum aperture is obtained while the pressure is almost constant across the fracture height which is consistent with the Perkins & Kern assumption (1961). The vector plot of water velocity shows small amount of leakoff into porous matrix resulting in a considerable decrease in pressure. Another factor decreasing the pressure inside fracture is the high compressibility and low viscosity of air interacted with the injected water which cannot be simulated in the models based on single phase fluid assumption. The numerical results show the capabilities of the developed numerical tool in the analysis of coupled hydro-mechanical multi-phase fluid flow through fractured and fissured rock and soil layers which are not available in the existing commercial codes based on the knowledge of the authors of this paper.

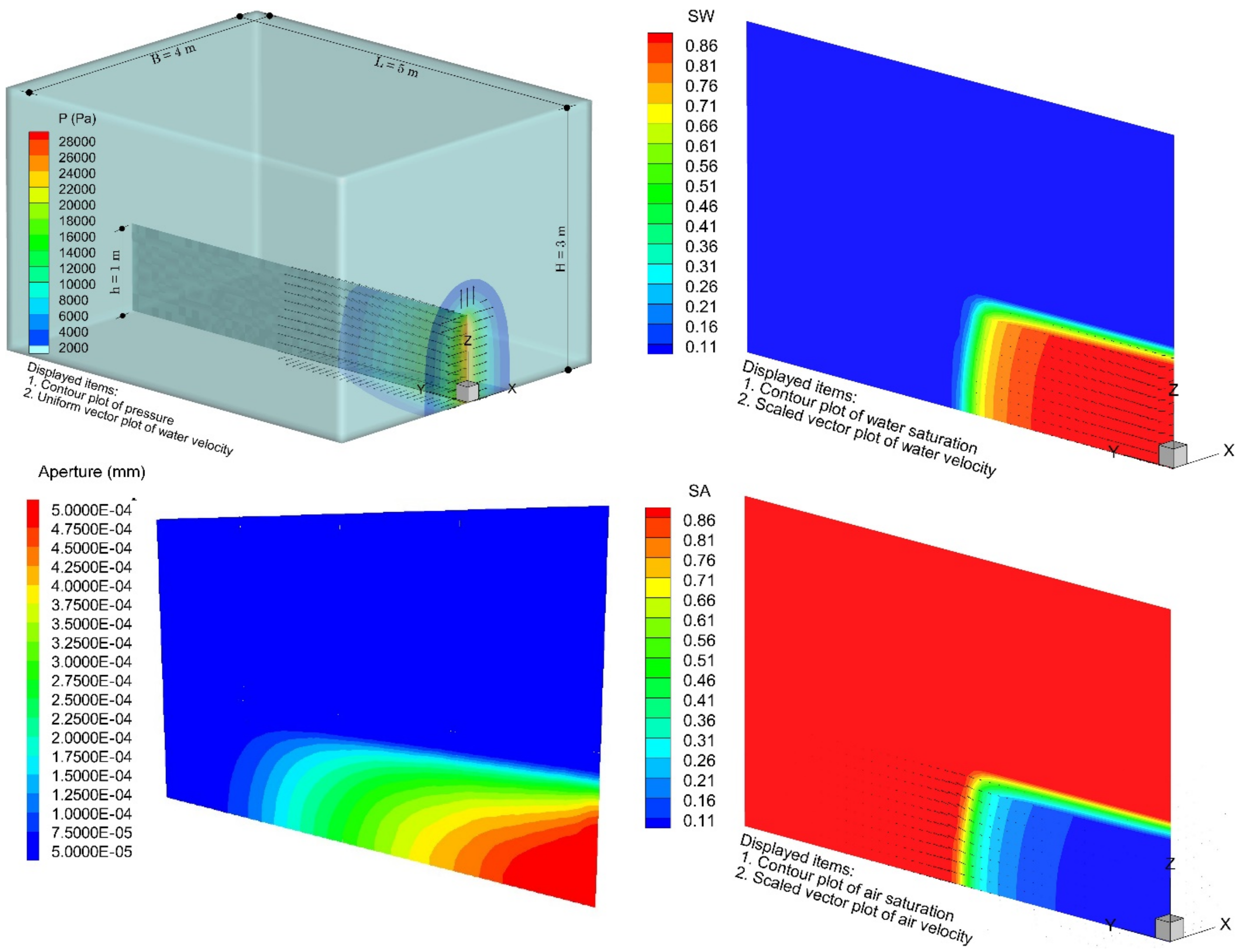

Figure 2. Simulation of air-water two phase flow through porous matrix containing a single fracture using the assumptions of Perkins and Kern (1961) for the fracture.

4 CONCLUSIONS

In this research, a computational tool has been developed by coupling FVBOS and *3DEC* software to simulate coupled hydro-mechanical multi-phase fluid flow through both fractures and pore spaces in fractured reservoir rock formations. The developed numerical tool takes a crucial step towards the better understanding of fluid flow through fractured porous media and it provides an effective tool for the assessment of the efficiency of hydraulic fracturing treatment, production performance and probable risks during these stages in naturally fractured unconventional reservoir rock formations.

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