Use of a fully tensorial approach to characterize the stress variability at Forsmark

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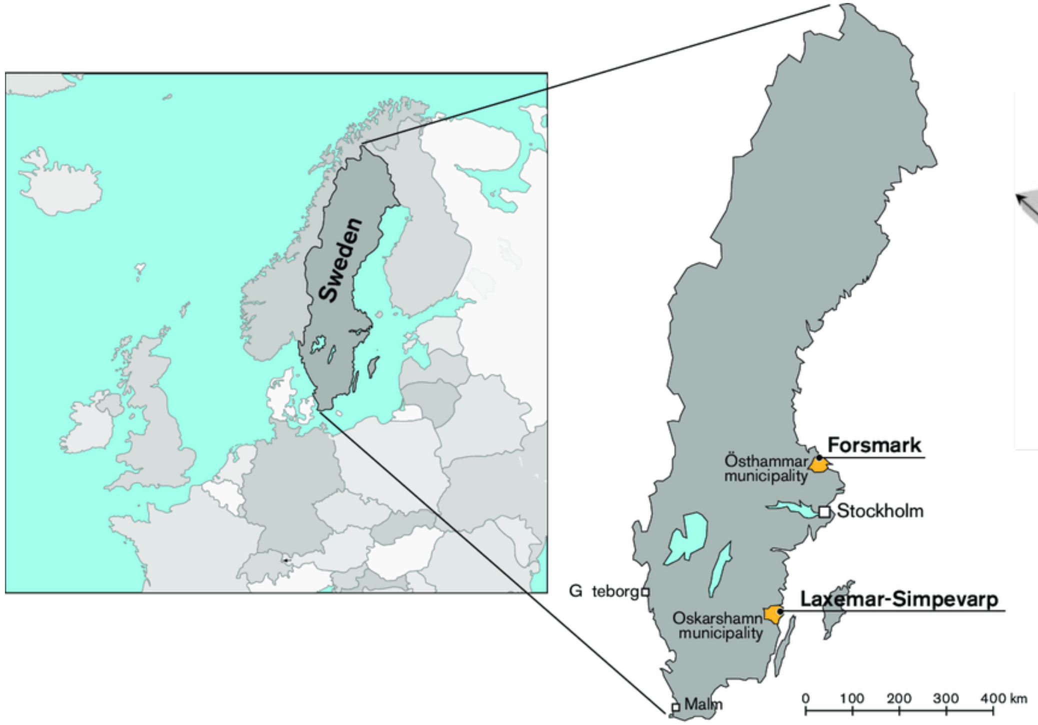
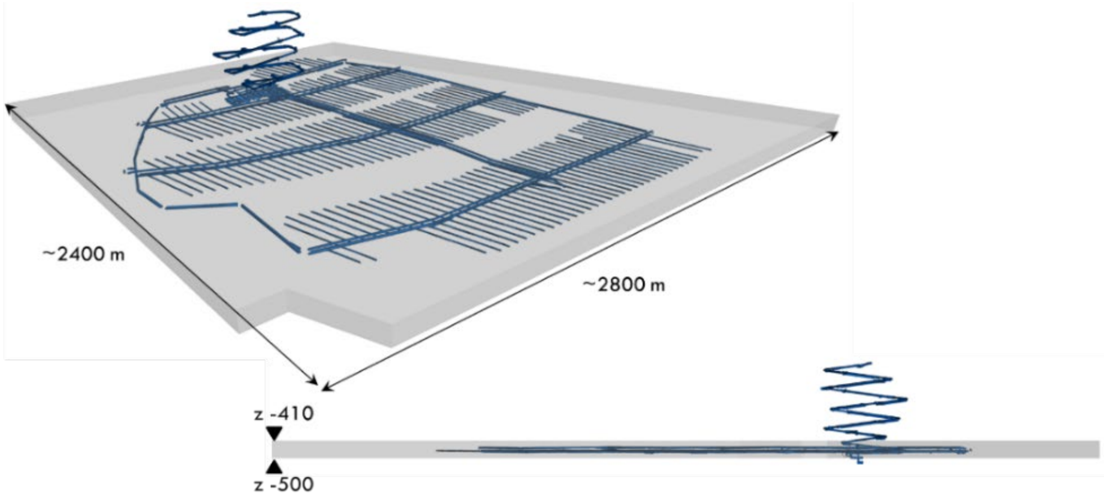
Motivation (1/3)

- The Forsmark site is the chosen location for the repository for spent nuclear fuel in Sweden.
- The stress magnitude and orientation is an important factor in the design of the repository.



Location of Forsmark



Layout of the repository

Motivation (2/3)

- Scalar/vector approach - based on the separate evaluation of magnitudes and orientations
- Tensorial approach – based on the evaluation of the principal stress magnitude and orientation, obtained from the mean stress tensor

A scalar/vector approach is incorrect, because it leads to non-perpendicular principal stresses, violating the principle of continuum mechanics

Depth [m]	Stress nr	Stress tensor components (MPa)					
		σ_{xx}	σ_{xy}	σ_{xz}	σ_{yy}	σ_{yz}	σ_{zz}
416.55	S_1	43.2	4.7	-3.4	32.7	-0.3	15.3
416.57	S_2	41.2	6.6	-3.3	31.3	0.5	17.7
416.60	S3	42.9	8.8	-4.0	35.8	2.8	14.6
416.62	S4	45.1	5.4	-4.4	31.6	2.3	18.3
416.68	S5	42.6	4.4	-1.9	28.3	0.8	15.1
...	...	...	...	...	...	...	...
417.17	S17	29.7	3.0	-4.9	40.5	-0.1	14.2

Stress database

Mean principal stress	σ_1 & σ_2 (°)	σ_2 & σ_3 (°)	σ_3 & σ_1 (°)
Scalar/vector approach	10	86	85
Tensorial approach	90	90	90

Angle between principal stress directions

Motivation (3/3)

- Tensorial approaches can be divided in two sub-groups: quasi-tensorial and fully tensorial.
  - Quasi-tensorial approaches are not adequate to assess the overall stress field dispersion, when the various stress tensor components are highly correlated.

	σ_{xx}	σ_{xy}	σ_{xz}	σ_{yy}	σ_{yz}	σ_{zz}
σ_{xx}	1.00	0.53	0.18	-0.67	0.01	0.42
σ_{xy}	0.53	1.00	0.08	-0.66	-0.12	0.48
σ_{xz}	0.18	0.08	1.00	-0.32	-0.29	-0.24
σ_{yy}	-0.67	-0.66	-0.32	1.00	0.03	-0.44
σ_{yz}	0.01	-0.12	-0.29	0.03	1.00	0.17
σ_{zz}	0.42	0.48	-0.24	-0.44	0.17	1.00

  - Fully tensorial approaches take into account the stress correlation, but to an accurate estimation of the correlation coefficients, a minimum number of 7 data in a limited depth range of 50-100 m, must be available.

Tensorial approach

- Normal distribution of distinct stress components
  - ❖ Quasi-tensorial: Univariate normal distribution
  - ❖ Fully tensorial: Multivariate normal distribution (Gao & Harrison, 2018)

$$f_{sd} = \frac{1}{\sqrt{(2\pi)^{\frac{1}{2}p(p+1)}|\Omega|}} exp\left(-\frac{1}{2}(s_d - m_d)^T(\Omega)^{-1}(s_d - m_d)\right),$$

Ω - covariance matrix

s_d - magnitude of the six distinct stress components

m_d - mean of the magnitude of the six distinct stress components

p – dimension of the stress tensor

- Effective variance: scalar value that measures the overall stress dispersion [MPa^2]

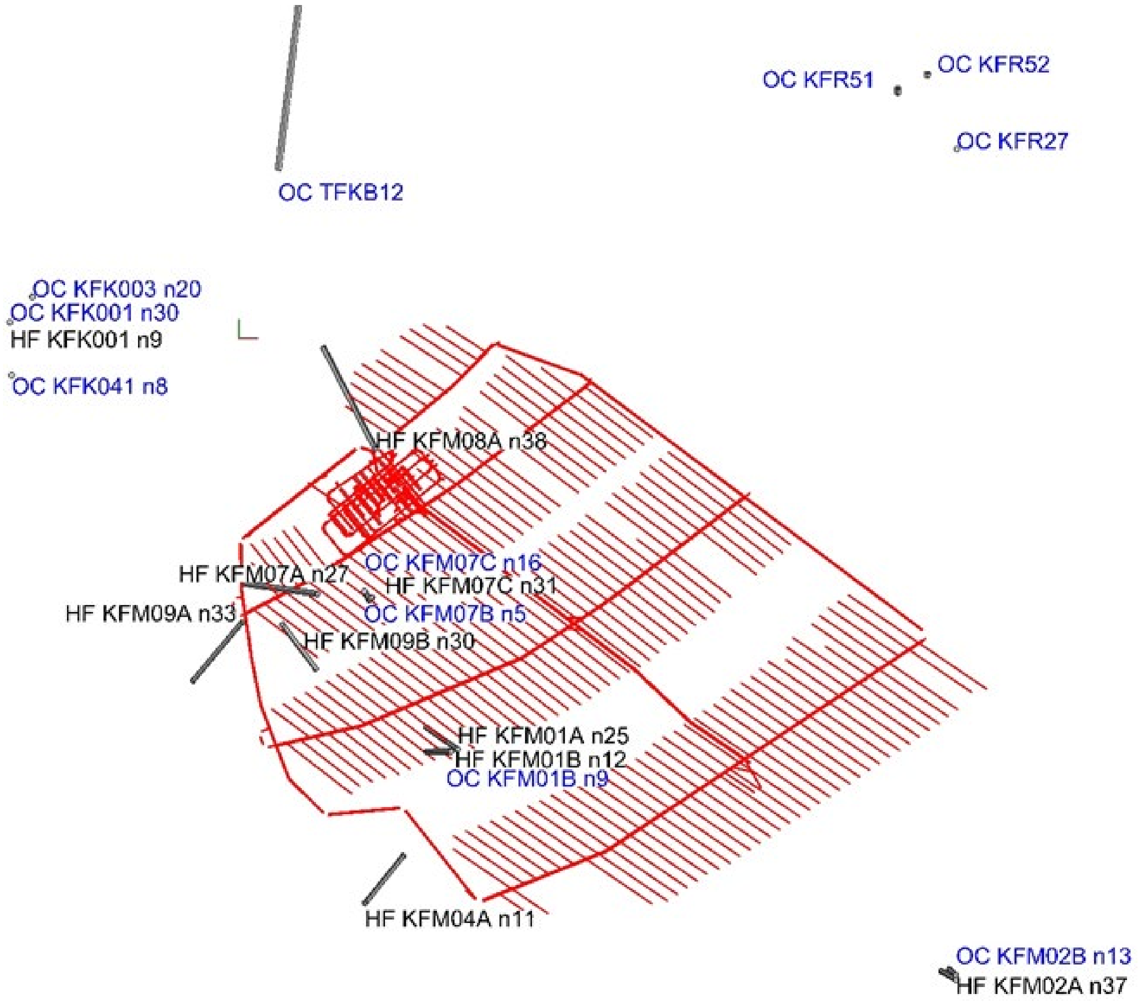
$$V_{eff} = \sqrt[\frac{1}{2}p(p+1)]{|\Omega|}$$

Gao, K. & Harrison, J. P. 2018. Multivariate distribution model for stress variability characterisation. Int. J. Rock Mech. Min. Sci., 102: 144–154.

Application to stress measurement data

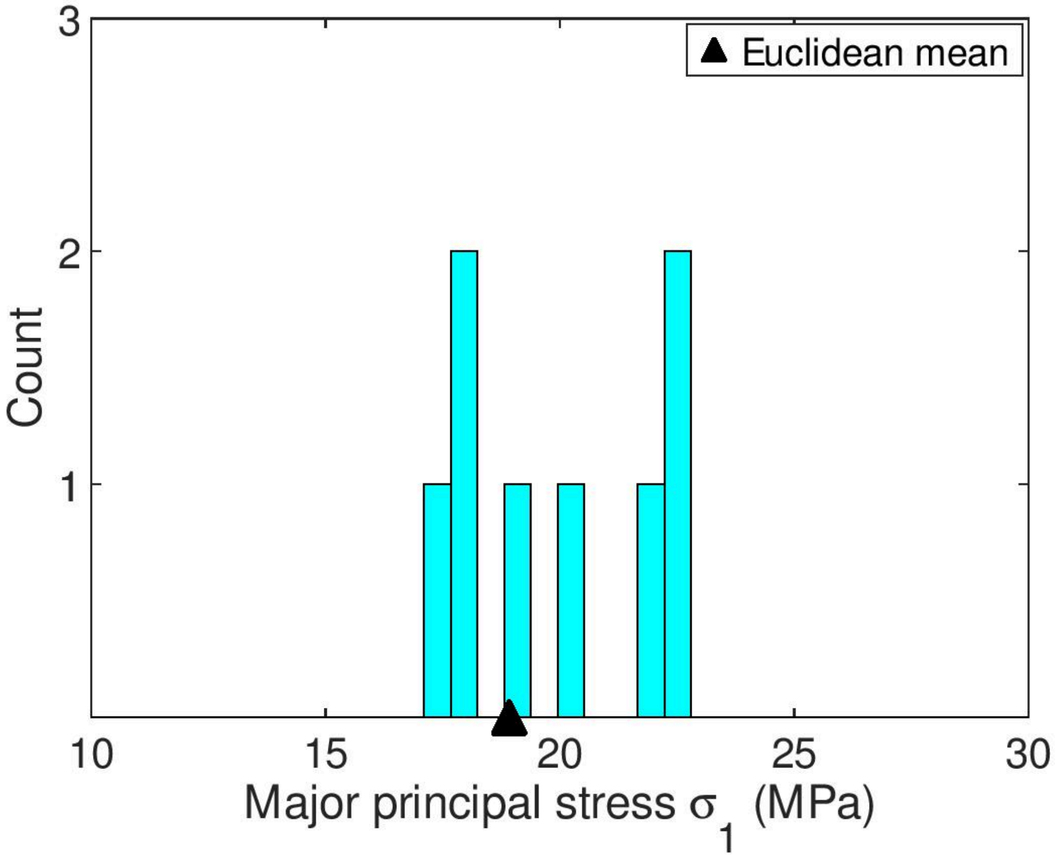
- Available data

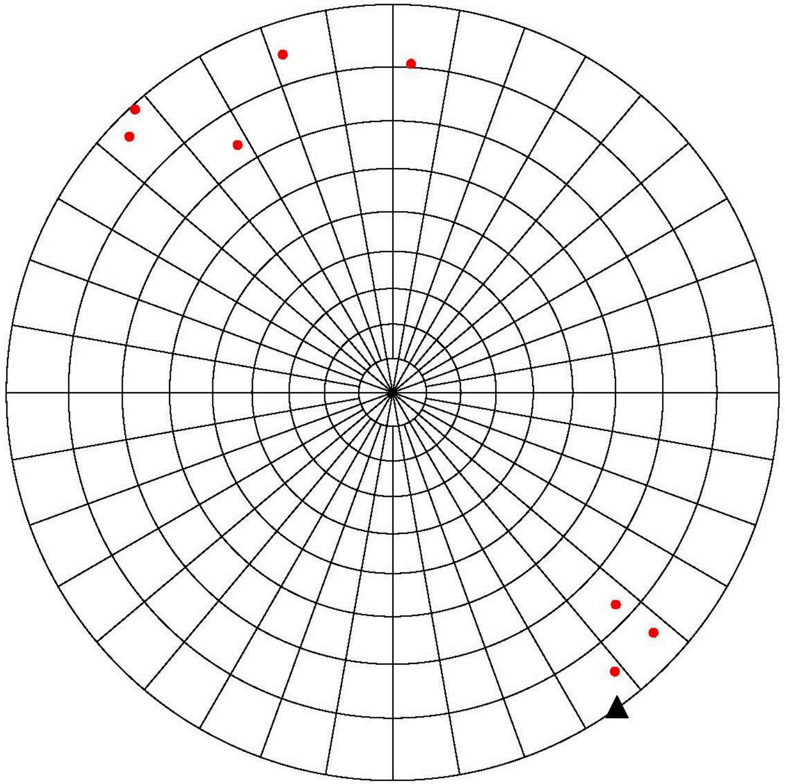
	Overcoring data	Hydraulic data
Number of boreholes	6	10
Number of measurement points	90	60
Depth range [m]	14-502	29-960



Results from stress measurement data (1/3)

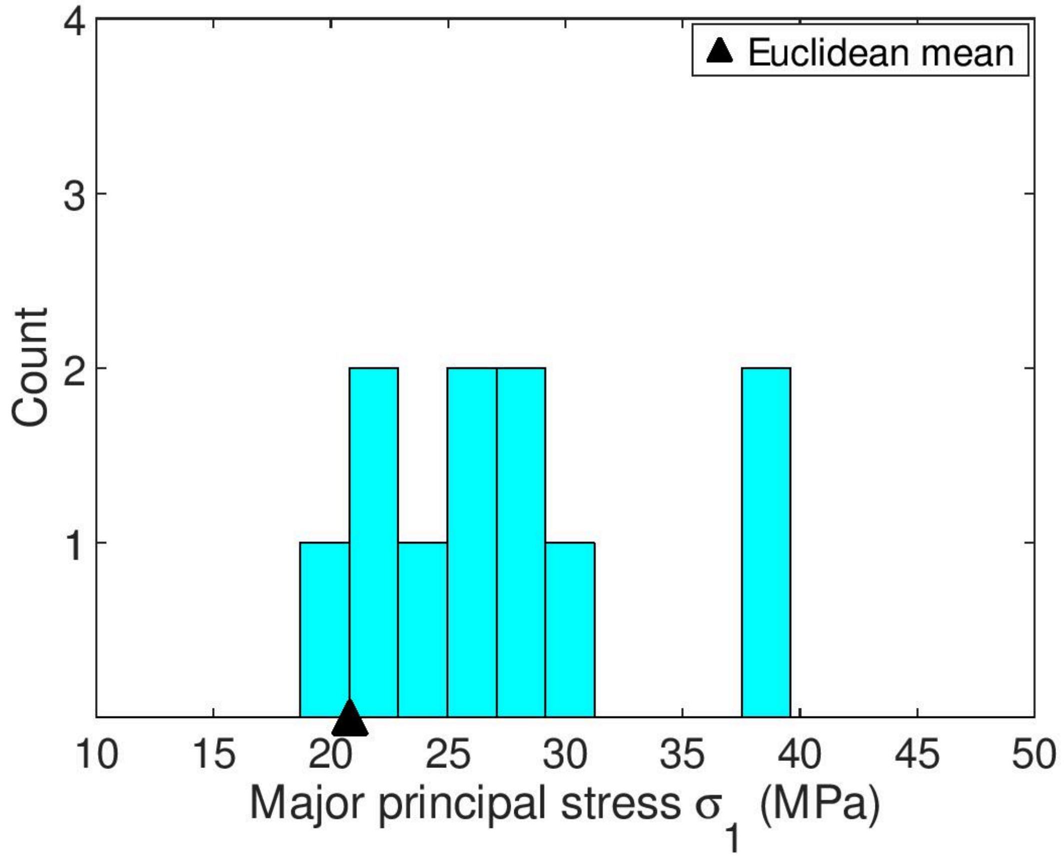
- Selected data from borehole DBT3 (Depth range: 104-155 m, number = 8)
  - ❖ Magnitude and orientation of the major principal stress

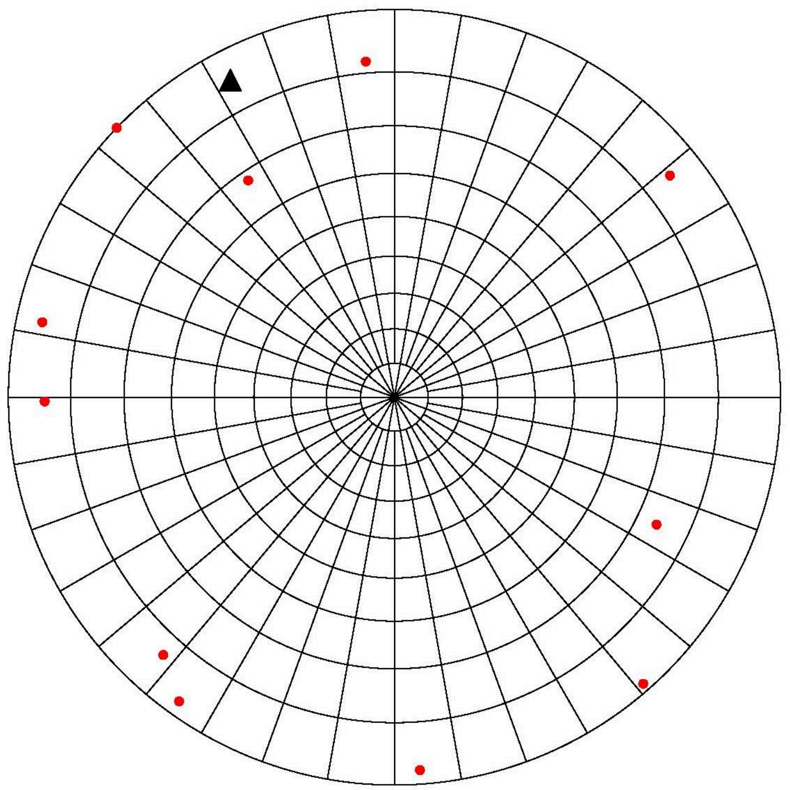




Results from stress measurement data (2/3)

- Selected data from borehole KFM07C (Depth range: 158-197 m, number = 11)
  - ❖ Magnitude and orientation of the major principal stress





Results from stress measurement data (3/3)

- Correlation coefficients (Borehole DBT3)

	σ_{xx}	σ_{xy}	σ_{xz}	σ_{yy}	σ_{yz}	σ_{zz}
σ_{xx}	1.00	-0.62	0.22	-0.17	0.61	0.12
σ_{xy}	-0.62	1.00	-0.04	0.56	-0.56	0.52
σ_{xz}	0.22	-0.04	1.00	0.20	-0.17	0.30
σ_{yy}	-0.17	0.56	0.20	1.00	-0.40	0.13
σ_{yz}	0.61	-0.56	-0.17	-0.40	1.00	0.16
σ_{zz}	0.12	0.52	0.30	0.13	0.16	1.00

Borehole DBT3

	σ_{xx}	σ_{xy}	σ_{xz}	σ_{yy}	σ_{yz}	σ_{zz}
σ_{xx}	1.00	0.47	0.05	0.10	0.12	0.24
σ_{xy}	0.47	1.00	0.36	0.45	0.29	0.31
σ_{xz}	0.05	0.36	1.00	0.31	-0.37	-0.02
σ_{yy}	0.10	0.45	0.31	1.00	0.15	0.42
σ_{yz}	0.12	0.29	-0.37	0.15	1.00	0.59
σ_{zz}	0.24	0.31	-0.02	0.42	0.59	1.00

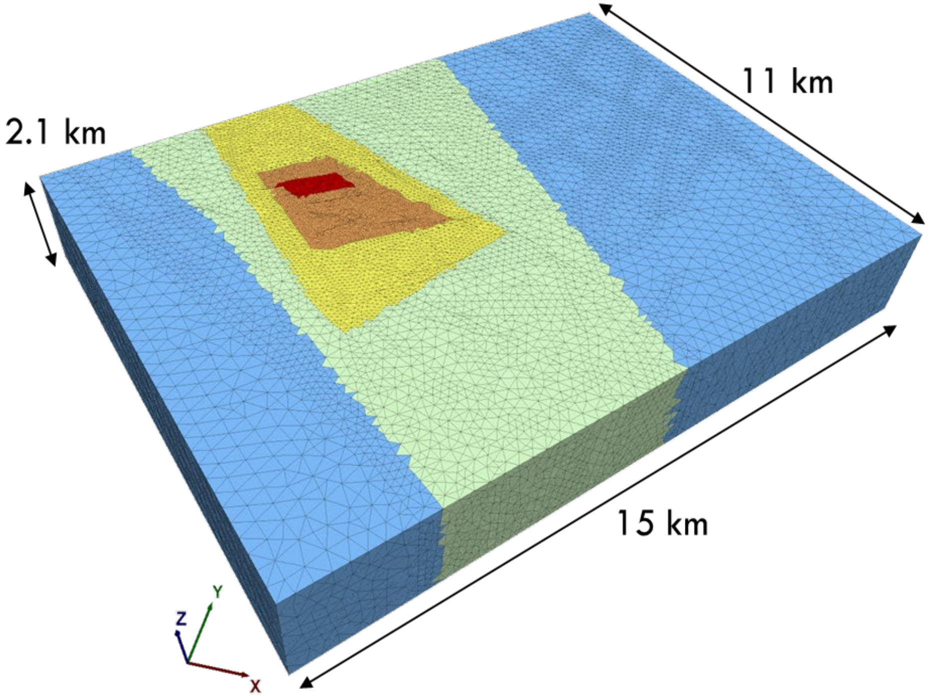
Borehole KFM07C

- Effective variance V_{eff} [MPa^2]

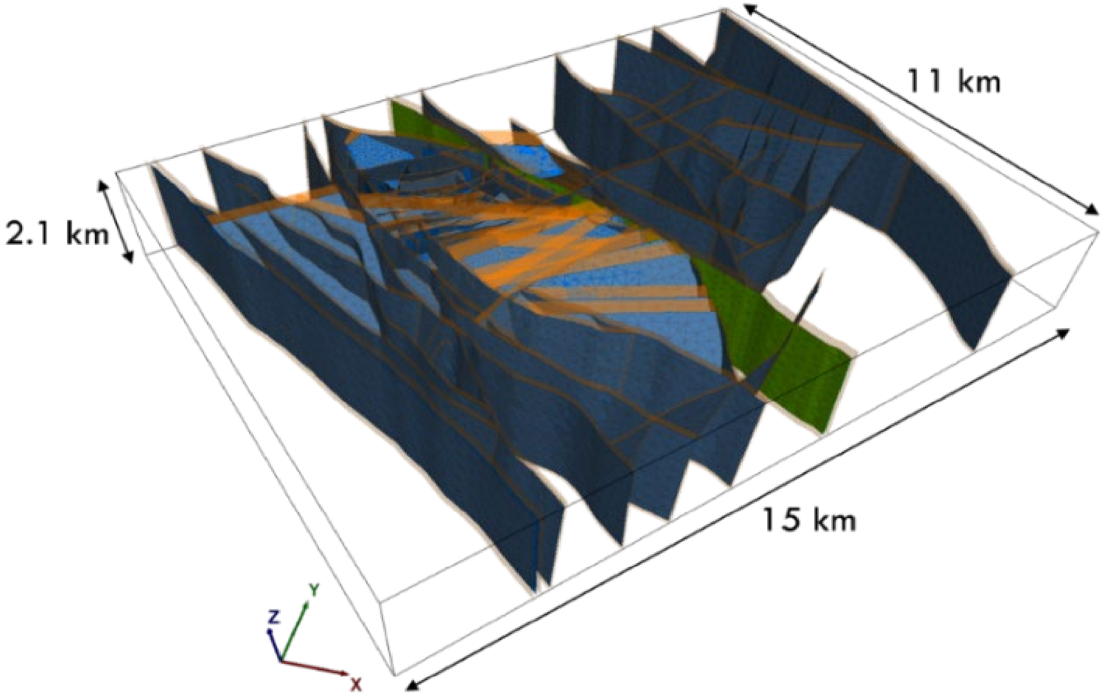
Borehole	Fully tensorial	Quasi-tensorial
DBT3	2.68	5.77
KFM07C	14.60	19.79

Application to the existing regional stress model (1/2)

- 3D regional stress model for Forsmark site



3DEC model

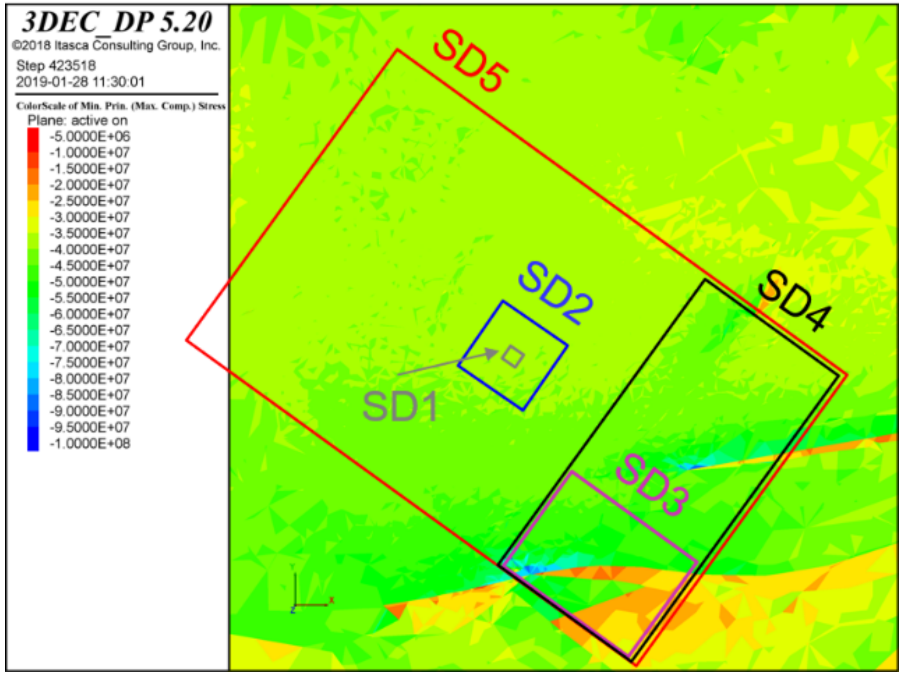


Undulating deformation zones represented by the blue, orange and green colors

Hakala, M., Ström, J., Valli, J., Juvani J. 2019. Stress-geology interaction modelling of the Forsmark site. Rock Mechanics Consulting Finland Oy.

Application to the existing regional stress model (2/2)

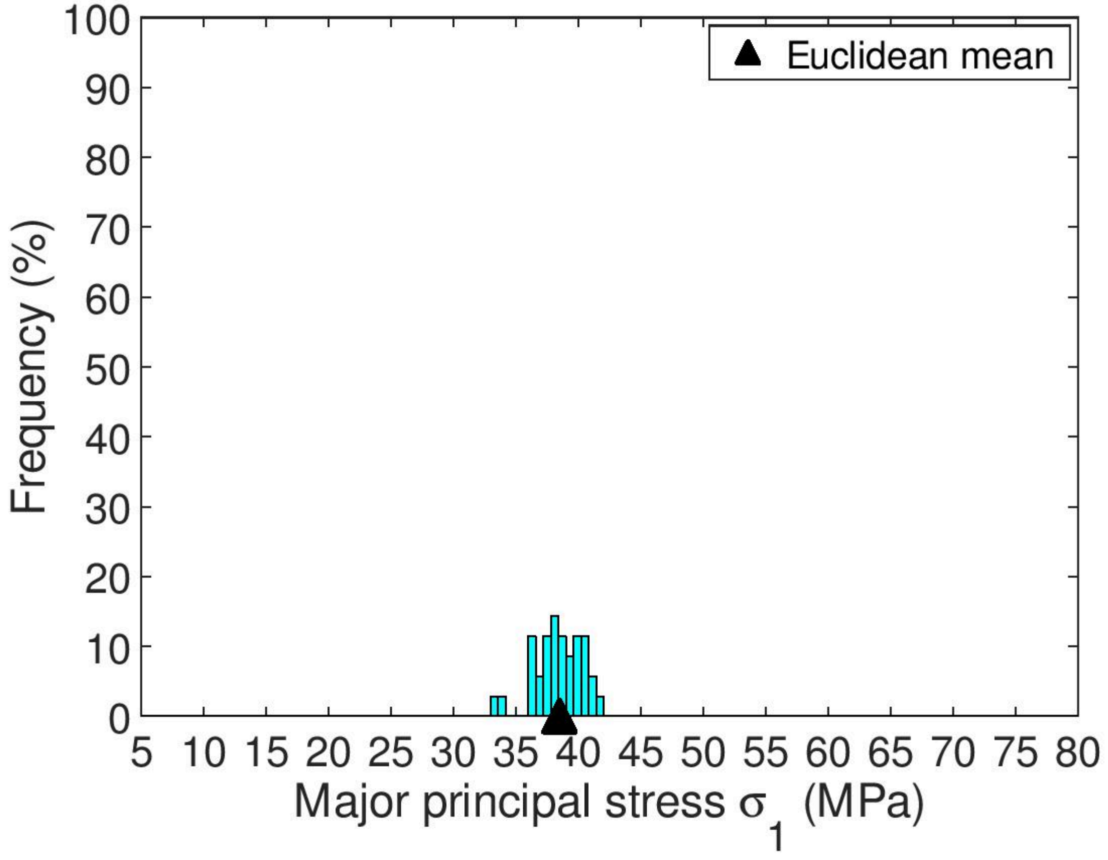
- Consideration of different sampling domains (SD) to evaluate the effects of the physical scale and number of data on the stress variability (slice with a thickness of 15 m)
  - SD1: drift scale (assumed stress homogeneity)
  - SD2: scale of hundreds of meters (assumed stress homogeneity)
  - SD3: scale of hundreds of meters (one region with stress heterogeneity identified in the contours of the principal stresses)
  - SD4: scale of hundreds of meters (two regions with stress heterogeneity identified in the contours of the principal stresses)
  - SD5: scale of the repository

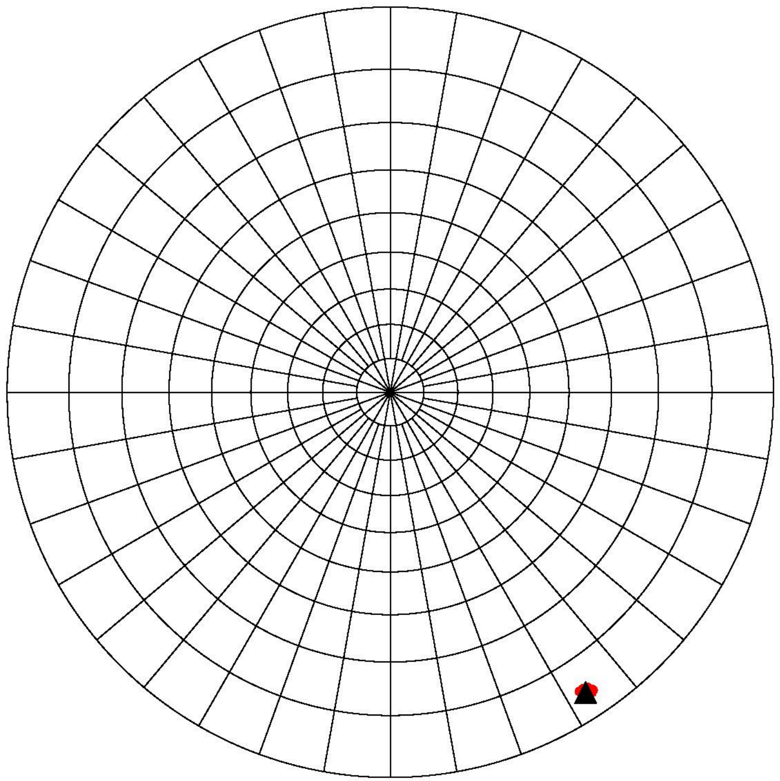


Major principal stress

Results from the existing stress model (1/7)

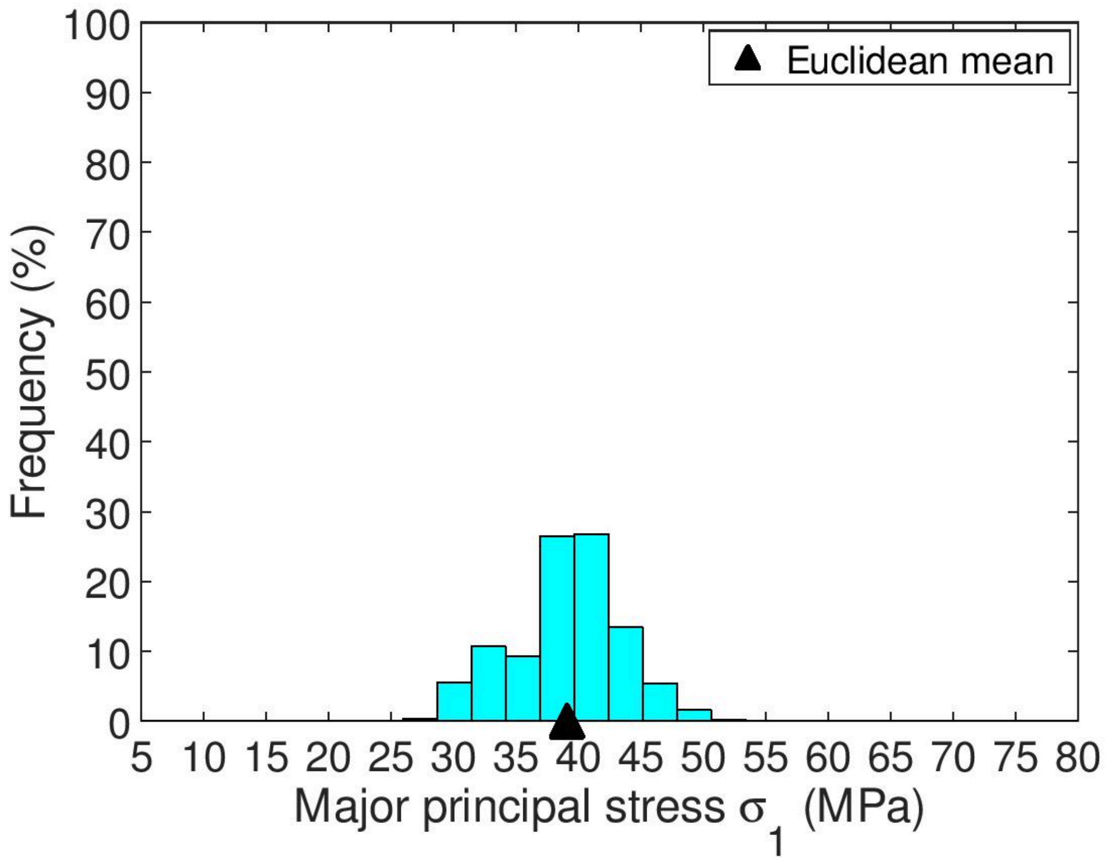
- Magnitude and orientation of the major principal stress –SD1

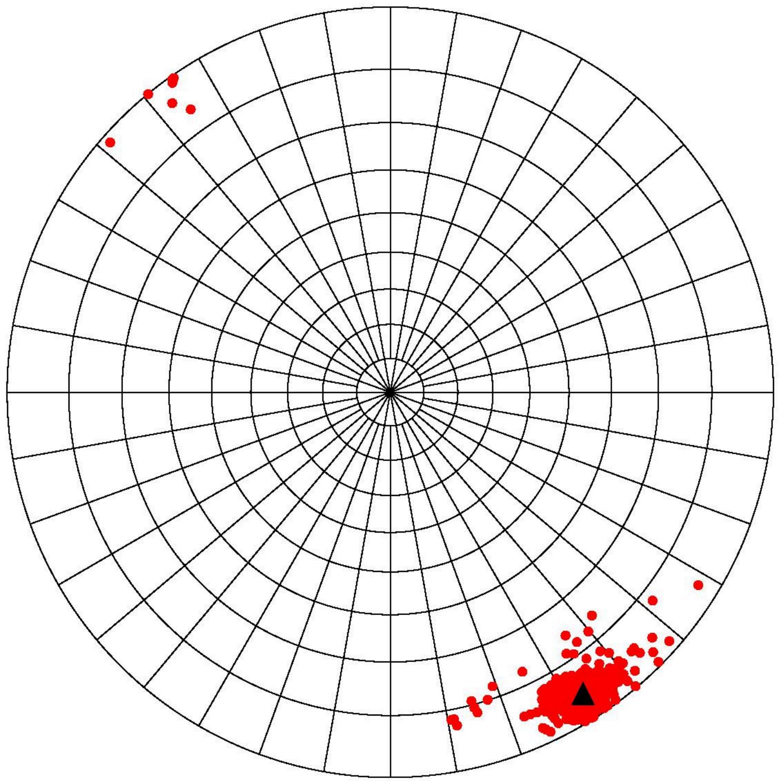




Results from the existing stress model (2/7)

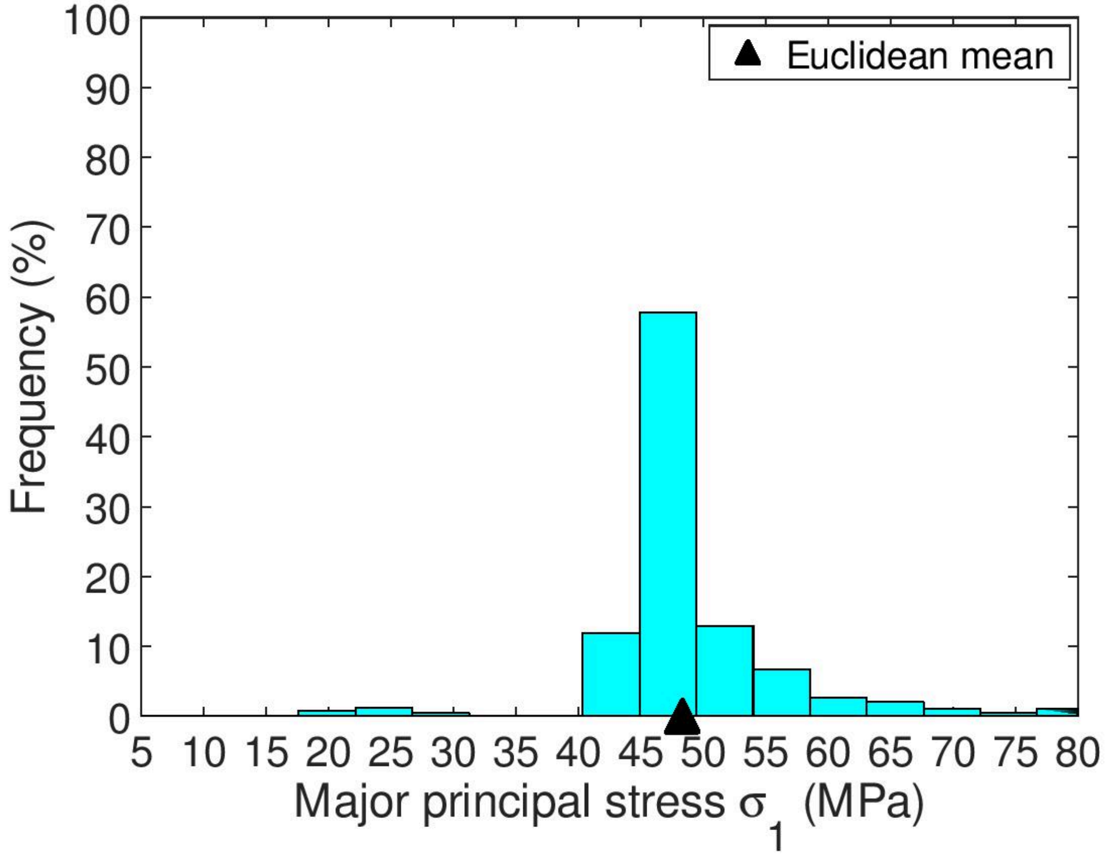
- Magnitude and orientation of the major principal stress – SD2

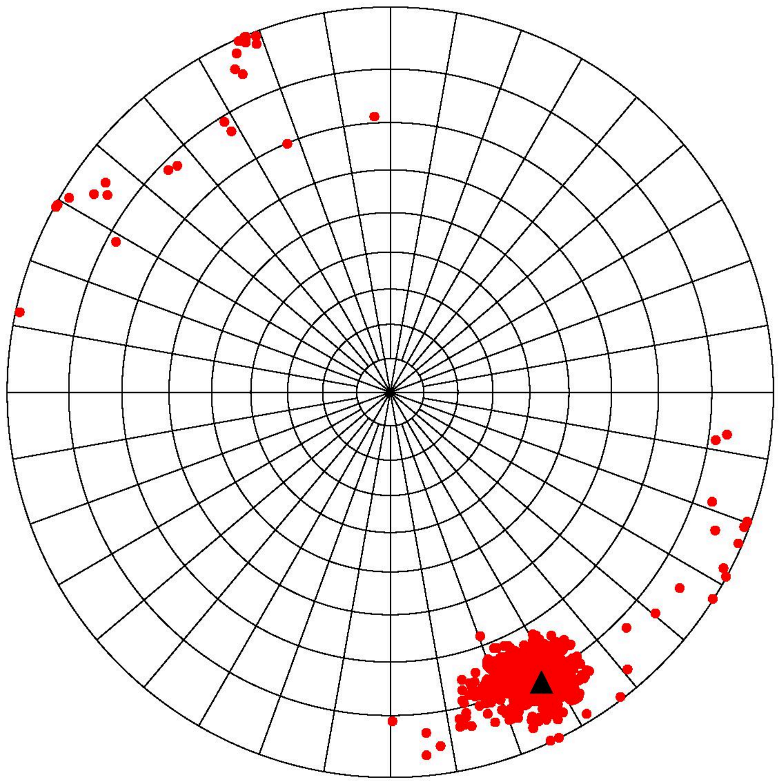




Results from the existing stress model (3/7)

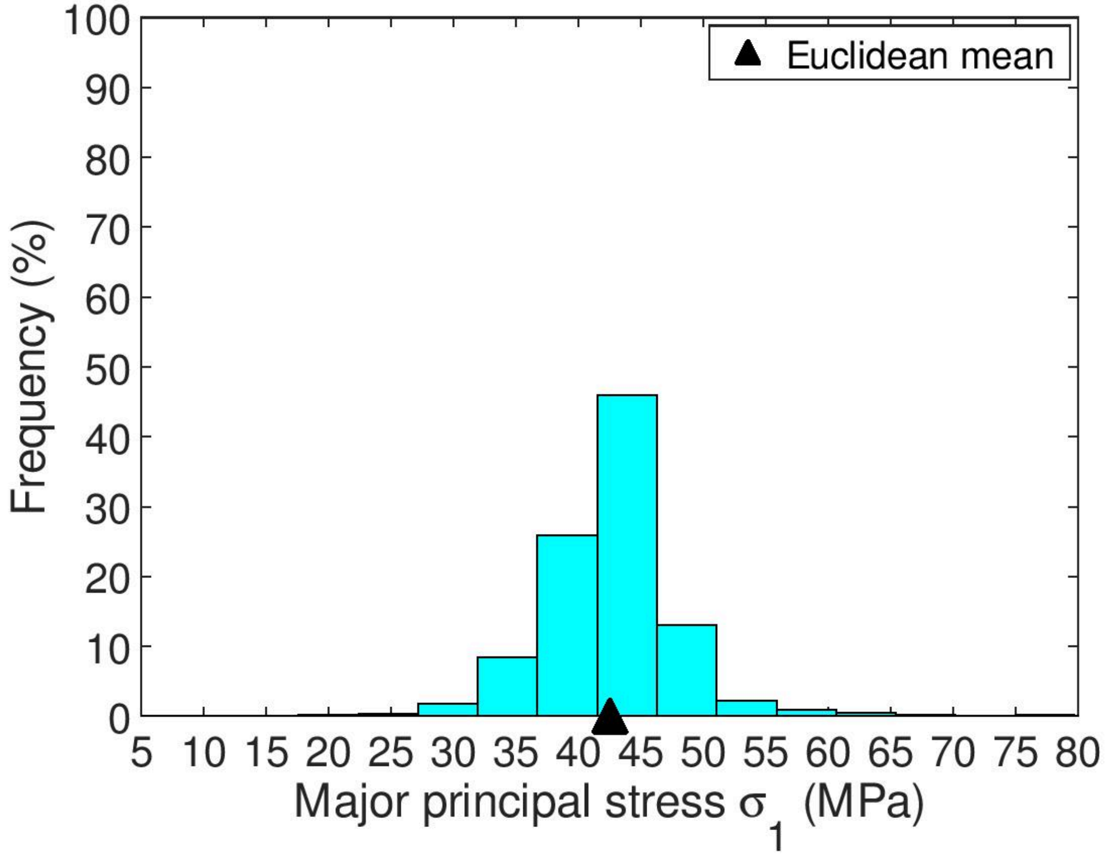
- Magnitude and orientation of the major principal stress – SD3

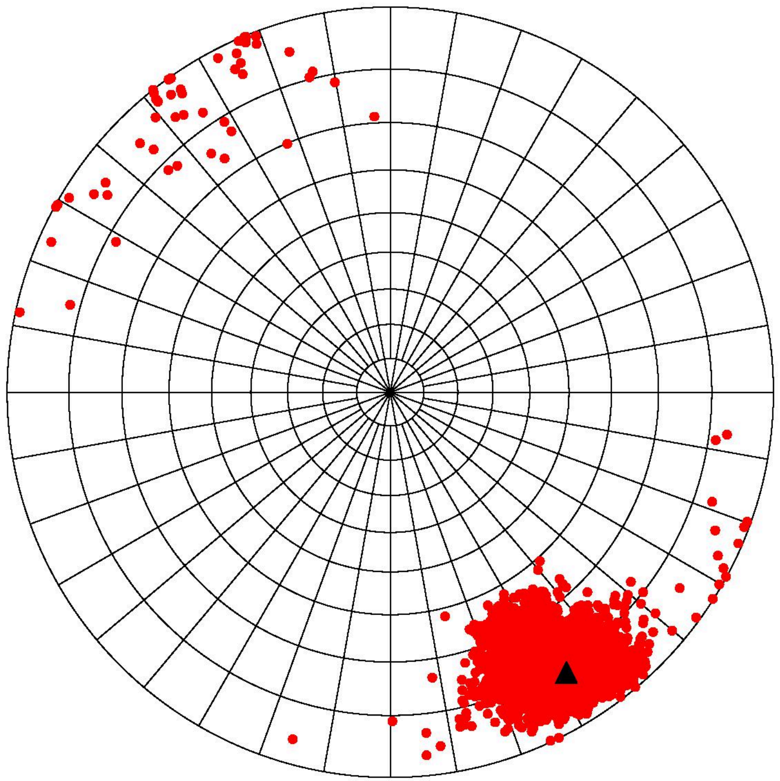




Results from the existing stress model (4/7)

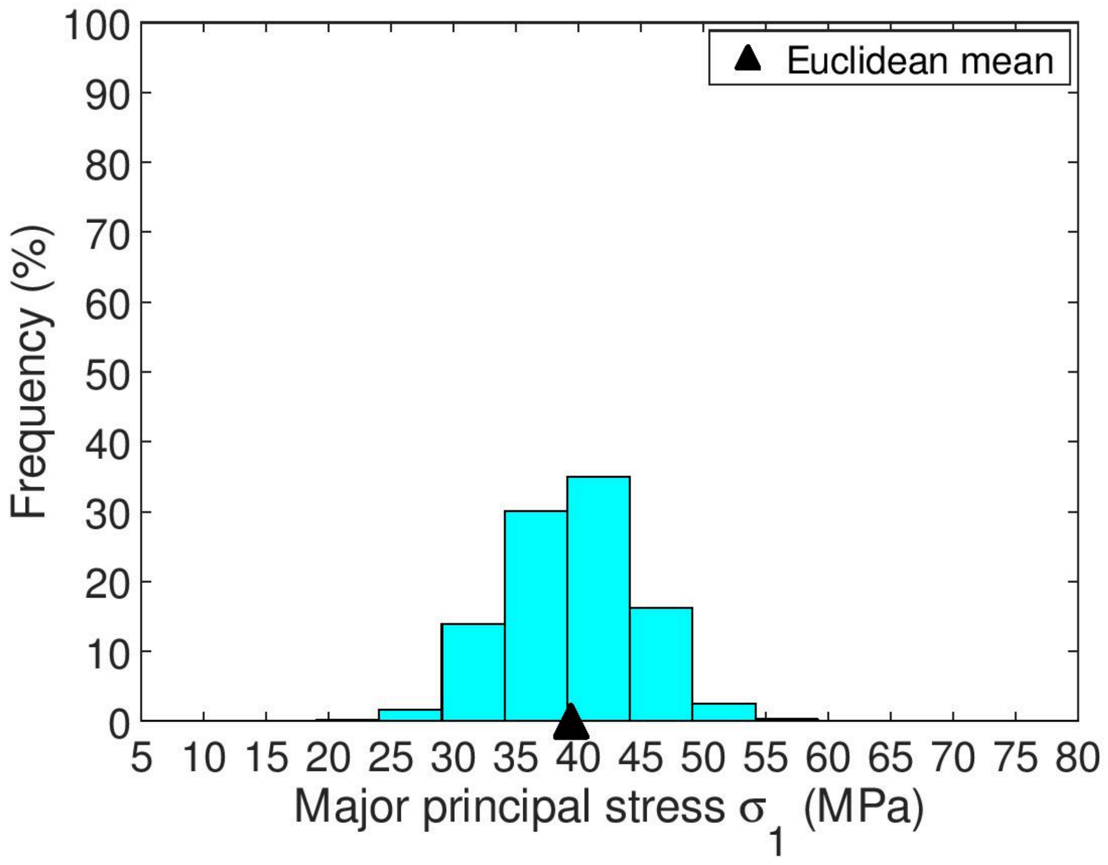
- Magnitude and orientation of the major principal stress – SD4

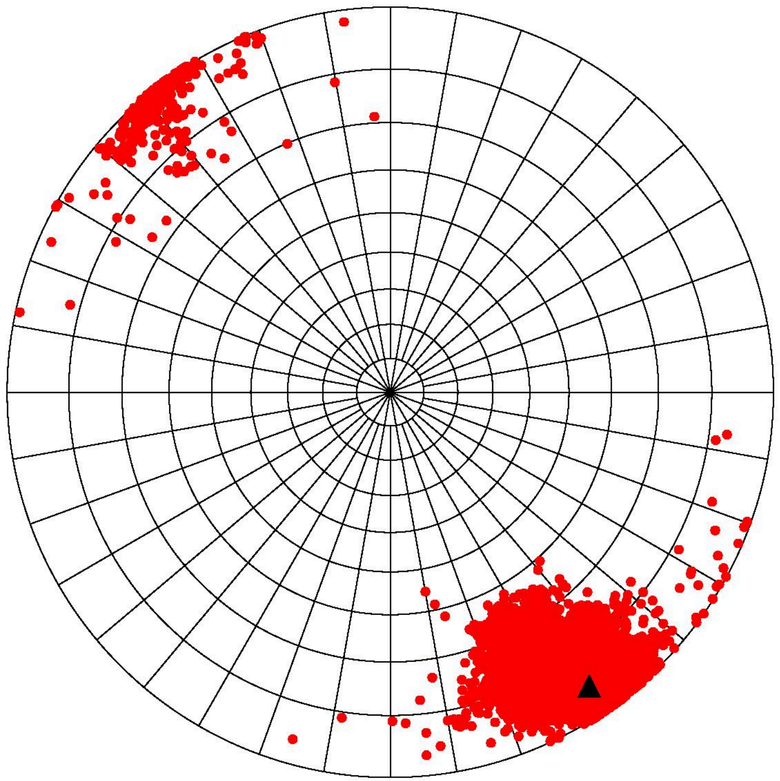




Results from the existing stress model (5/7)

- Magnitude and orientation of the major principal stress – SD5





Results from the existing stress model (6/7)

- Matrix of the correlation coefficients

	σ_{xx}	σ_{xy}	σ_{xz}	σ_{yy}	σ_{yz}	σ_{zz}
σ_{xx}	1.00	-0.80	-0.29	0.96	-0.62	0.97
σ_{xy}	-0.80	1.00	0.23	-0.79	0.43	-0.71
σ_{xz}	-0.29	0.23	1.00	-0.37	-0.36	-0.27
σ_{yy}	0.96	-0.79	-0.37	1.00	-0.54	0.95
σ_{yz}	-0.62	0.43	-0.36	-0.54	1.00	-0.61
σ_{zz}	0.97	-0.71	-0.27	0.95	-0.61	1.00

Sampling domain SD2
(stress homogeneity)

	σ_{xx}	σ_{xy}	σ_{xz}	σ_{yy}	σ_{yz}	σ_{zz}
σ_{xx}	1.00	-0.63	-0.21	0.70	0.22	0.45
σ_{xy}	-0.63	1.00	0.51	-0.62	-0.33	0.05
σ_{xz}	-0.21	0.51	1.00	-0.44	-0.52	0.34
σ_{yy}	0.70	-0.62	-0.44	1.00	0.54	0.00
σ_{yz}	0.22	-0.33	-0.52	0.54	1.00	-0.22
σ_{zz}	0.45	0.05	0.34	0.00	-0.22	1.00

Sampling domain SD3
(stress heterogeneity)

Results from the existing stress model (7/7)

- Overal stress field dispersion

SD	Effective variance V_{eff} [MPa^2]			
	Depth (m)			
	300	450	470	500
SD1	0.02	0.04	0.04	0.01
SD2	0.48	0.44	0.45	0.49
SD3	5.30	6.60	4.43	4.31
SD4	7.46	3.88	2.82	2.56
SD5	3.53	1.82	1.72	1.65

Fully tensorial approach

SD	Effective variance V_{eff} [MPa^2]			
	Depth (m)			
	300	450	470	500
SD1	0.10	0.26	0.27	0.08
SD2	2.74	1.91	1.84	2.01
SD3	16.68	14.51	7.30	6.76
SD4	13.58	7.58	5.10	4.76
SD5	6.63	4.63	4.32	4.12

Quasi-tensorial approach

Conclusions

- The various stress tensor components are significantly correlated.
- The stress dispersion obtained with a quasi-tensorial approach is overestimated and hence, not exclusively related with the stress heterogeneity.
- When the fully tensorial approach is applied to stress measurement data, the division in several data sets or a large number of data may be needed.
- When the fully tensorial approach is applied to numerical modelling results:
  - A large number of stress data at any depth can be considered;
  - The stress dispersion of several sampling domains and depth intervals can be quantified;
  - It can assist in the design of future stress measurement campaigns; and
  - The overall stress dispersion is smaller than that obtained from data analysis.

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